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ABSTRACT

These training materials are designed to stress the importance of a close relationship between concepts and skills when teaching mathematics to young children, to present material on the important area of problem-solving, and to encourage adults to use a wide range of appropriate techniques in evaluating their work and children's work in mathematics. The materials include these six workshop sessions: (1) concept development; (2) skill development; (3) hand-in-hand: concepts and skills (reviewing the interrelated nature of concept and skill development, followed by exploration of materials and methods for enhancing development of selected mathematical ideas; (4) problem-solving; (5) evaluation; and (6) make it, take it (creating materials for classroom use). Each of the first three sessions might be followed by a short work session in which participants make materials for use in their classrooms. Each workshop (which requires 2 to 3 hours) features lecture and discussion segments. Some sessions also include time for examining materials and problem-solving. Many masters for making overhead projector transparencies for workshop leaders and handouts for participants are included. Workshop sessions cover most content areas for the Mississippi Test of Basic Skills which is given in grade 3 (and again in grades 5, 8, and 11). (JN)

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INSTRUCTIONAL TELEVISION FOR INDEPENDENT STUDY

MATERIALS COMPONENT OF CYCLE 3 CONTRACT

TEACHER/ASSISTANT TEACHER STAFF DEVELOPMENT MATERIALS

SE 041 218 ED 265 018

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INSTRUCTIONAL TECHNIQUES
FOR
INCREASING THE MATHEMATICS COMPETENCIES
OF
YOUNG CHILDREN

Training Materials for Teachers and Assistant Teachers

ACKNOWLEDGEMENTS

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INTRODUCTION

The following information will assist persons in presenting a workshop that will give teachers and assistant teachers an overview of techniques with regard to mathematics instruction. The intent of the State Department of Education is to provide resource material that would enable a school or school district to provide one or more inservice programs on this topic.

Before beginning the session, the inservice leader should review the information and concepts to be presented. There are suggested activities for each of the concepts presented.

INSTRUCTIONAL TECHNIQUES FOR INCREASING THE MATHEMATICS COMPETENCIES OF YOUNG CHILDREN

INSTRUCTIONAL TECHNIQUES FOR INCREASING THE MATHEMATICS COMPETENCIES OF YOUNG CHILDREN

These training materials have been designated for use with teachers and assistant teachers of young children. The materials are designed to stress the importance of a close relationship between concepts and skills when teaching mathematics to young children, to present material on the important area of problem solving, and to encourage adults to use a wide range of appropriate techniques in evaluating their work and children's work in mathematics.

The materials include the following six workshop sessions:

Concept Development

Skill Development

Hand-in-Hand: Concepts and Skills

Problem Solving

Evaluation

Make It, Take It (Creating materials for classroom use).

Each of the first three sessions might be followed by a short work session in which participants make materials for use in their classrooms. Each session features lecture and discussion segments. Some sessions also include time for examining materials and problem solving.

To help workshop leaders, many masters for making transparencies for the overhead projector are included. These transparencies will help leaders to present and summarize major points and to illustrate examples. Some masters for handouts for participants are included as are patterns for making teaching-learning materials.

Leaders should note that the workshop sessions cover most content areas for the Mississippi Test of Basic Skills which will be given in the third grade (as well as the 5th, 8th and 11th grades). Dealing with these skills and the concepts that underlie them will be the responsibility of **every** teacher and assistant teacher, not just those in the grades where children will be tested. Hopefully, the use of pertinent examples of content that children should master will motivate educators to see that the workshop sessions are informative and pertinent.

Each workshop will require two to three hours depending on the length of time for involvement of the participants. The workshops could be spread through the school year or concentrated in preservice training sessions.

CONCEPT DEVELOPMENT

In this workshop session participants will receive information on the importance of concept development and how to teach concepts. Participants will give examples of concepts, work with materials they could later use to develop concepts with children, and suggest strategies for enhancing concept development in young children. The workshop leader should:

1. Define concepts, give examples, and encourage participants to give examples.
2. Discuss the importance of concept development.
3. Illustrate selected concepts and how to work with children to develop the concepts.
4. Facilitate group work in which participants brainstorm, develop, and share ideas for enhancing children's understanding of concepts.
5. Prepare the following materials prior to the presentation:

   Transparencies C-1 through C-11.

   Made-up cut-outs (from patterns) C-13, C-14, C-16 through C-19.

   Manipulative materials — 12 pencils, 12 clean empty softdrink cans and a box in which to put these, 8 margarine cups, 40 lima beans.

   Centimeter square graph paper (C-15). (½ sheet per participant).

   Handout sheets C-12.

   If the workshop leader cannot prepare the materials, perhaps a teacher or assistant could do so, and then keep the materials for classroom use. For this workshop, it is important that the leader and participants actually use materials. This is the behavior desired in the classroom and the leader should model the appropriate behavior! Leaders may also want to supply ditto masters or blank paper for groups to record their concept development ideas for future duplicating and distribution. Leaders may want to provide patterns C-13, C-14, and C-15 through C-20 to interested participants. The leader will also need to secure an overhead projector for the workshop.

1. What are concepts? (Use Transparency C-1.) Concepts are ideas that help people to think, express thoughts, and apply relationships to new situations and problems. Concepts help people deal with the meanings of mathematical operations, geometric and measurement relationships, and numbers. Concepts are also important in intelligent problem solving.

   People learn concepts through exploration of examples and nonexamples. Concepts develop over a period of time, so many different examples can be presented, discussed, and worked with. To understand a work like rectangle, children need to see, feel, and draw many different rectangular shapes. They also must be presented with some shapes that aren't rectangles. (Use Transparency C-2, C-3)

   Let's look at another concept — the meaning of a number such as 5. Children need to be exposed to lots of arrangements of five actual objects. (Use Transparency C-4, C-5.) If they always see 5 objects arranged in a similar fashion, they may focus on the pattern rather than the number. Teachers should encourage the children to create different arrangements of objects to illustrate five.

   Once children have seen different arrangements of similar objects, they need to make sets of 5 things that aren't exactly alike. Children can cut and paste pictures to illustrate five. They might draw five objects on a large numeral five as a further illustration of the meaning of the number five (Transparency C-6).

   Important concepts such as the meaning of multiplication should be approached in many ways. Multiplication has many models to fit different physical situations (Use Transparencies C-7, C-8). The Array model uses rows and columns in a rectangular arrangement. The Repeated Addition model involves a given number of sets, each set having the same

number of members. A less used but very interesting model known as the Cartesian Products or Combinations model pairs each member of one set with each number of another set. (Since this model is less familiar than the others to most people, also use the following problem.)

> An ice cream store has sugar cones and plain cones. They have 4 flavors of ice cream — chocolate, vanilla, strawberry, and grape. How many different single-dip cones can they make?

(Have participants draw and discuss their answers. Check answers with Transparency C-9.)

Use of each model gives meaning to the process of multiplication. Eventually we want children to be able to quickly give an answer to a problem such as 3 x 5, but if children do not understand the meaning of the process, they won't really know what their answers mean and they won't be able to use multiplication intelligently in problem solving.

(Next, have participants give examples of some other mathematics concepts children need to understand. Encourage discussion of what each concept means and examples and nonexamples of each concept.)

2. Why is concept development so important? (Transparency C-10.) Concept development enhances children's abilities to think for themselves. Without concept development, children are dependent on others to tell them what to do in problem solving situations. Children who have good understandings of mathematical concepts can apply their understandings to new situations.

Concept development is important because it adds meaning to mathematical operations, examination of relationships, and application of measurement ideas. It avoids "cookbook type" procedures like "Well, you move this number up and cross this out . . ." that are heavily dependent on memorization of rules which may make little sense to young children.

Taking time to develop concepts by teaching for meaning actually saves time in the long run. Studies have shown that children who have had time and opportunity for concept development generally remember material better than those who have spent 75-90% of their time on drill and practice. The children also transfer ideas more effectively in new problem situations. In other words, they are able to use what they have learned.

Many studies have shown that classes spending about 75% of their time on "developmental activities" — those that build new knowledge on what the children already know and stress development of meaning through active involvement of children — score higher on achievement tests than do children who spend 75% of the class time on drill-related activities. Therefore, time spent on developing ideas and enhancing meaning pays off in the long run in several different ways.

3. How can teachers and assistants work with children to enhance concept development? Along with work on examples and nonexamples of concepts, children should explain and talk about what they are doing (teachers should welcome several different explanations), work with pictorial representations, handle manipulatives, use related games and puzzles, and draw or create their own representations. (Transparency C-11.)

We will explore some possible activities for the development of a concept. Since we have recently used the meaning of multiplication, let's examine various learning activities in this area using the handout sheet (C-12).

4. Group work to apply information given on concept development. Divide participants into groups of three or four. Assign a concept to each group, or let each group pick a different concept they will be teaching in the near future. Suggested topics are: meaning of fractions; telling time; regrouping ones and tens; comparing numbers; or identifying polygons such as squares, rectangles, triangles and circles.

Direct each group to:

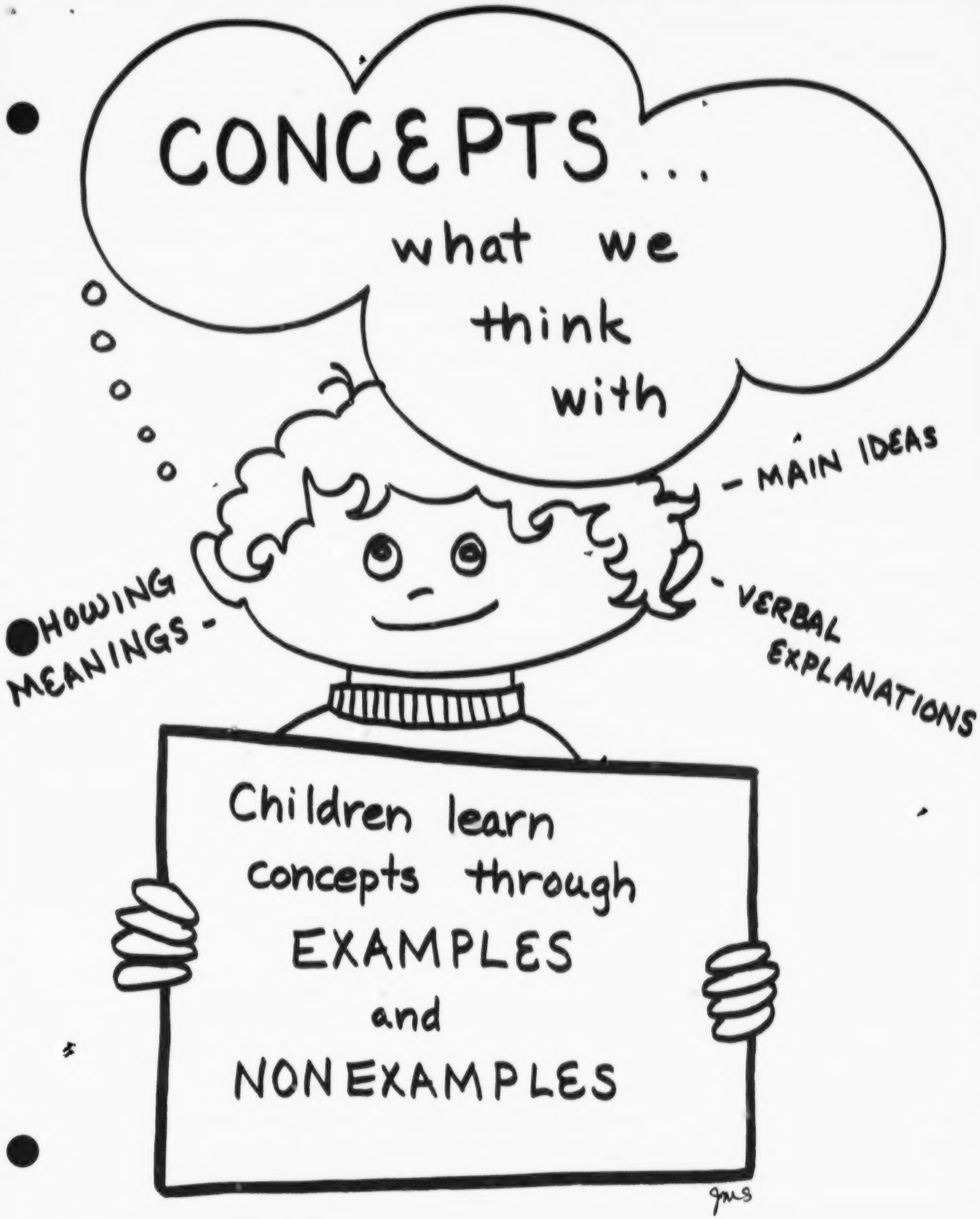
(a) Discuss one or two main ideas the children must master.

(b) Discuss examples and nonexamples of the concept and appropriate materials for presenting the examples and nonexamples.

(c) Develop several activities or problems where children could be actively involved in dealing with the concept. (Review some possible modes for activities from Transparency C-11.)

(d) Devise a method of briefly showing their ideas to the total group. If groups want to sketch ideas on paper or a chalkboard, others often appreciate this visual mode of presentation.

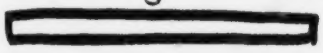
Let groups share their results. It may be possible for groups to write notes for later duplication and distribution to everyone. As a wrap-up activity, ask participants to review some of the modes in which children can deal with concepts. Leaders may also want to duplicate patterns for distribution to interested teachers assistants. Included in this publication are plans for a **Make It, Take It** session. Teachers and assistants may wish to choose patterns to make at this session.


CONCEPTS...
what we think with
- MAIN IDEAS
- VERBAL EXPLANATIONS
SHOWING MEANINGS -
Children learn concepts through EXAMPLES and NONEXAMPLES

Developing the Concept RECTANGLE

"Here's a rectangle."

"This one is a rectangle too."



"Here's another rectangle. Trace around it with your fingers."

"Try this one too."

"Tell me what's special about a rectangle."

"Is this a rectangle?"

"How 'bout this?"

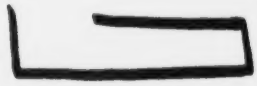
"Draw (or make) some rectangles for me." (Use the chalkboard, arrange yarn on a flannel board, trace in sand, mold clay.)

"Let somebody else trace around (see, feel) your rectangle."

"Show me some big rectangles... some small ones... some long skinny ones."

EXAMPLES and NONEXAMPLES of RECTANGLES

"Now watch ... this one is not a rectangle"

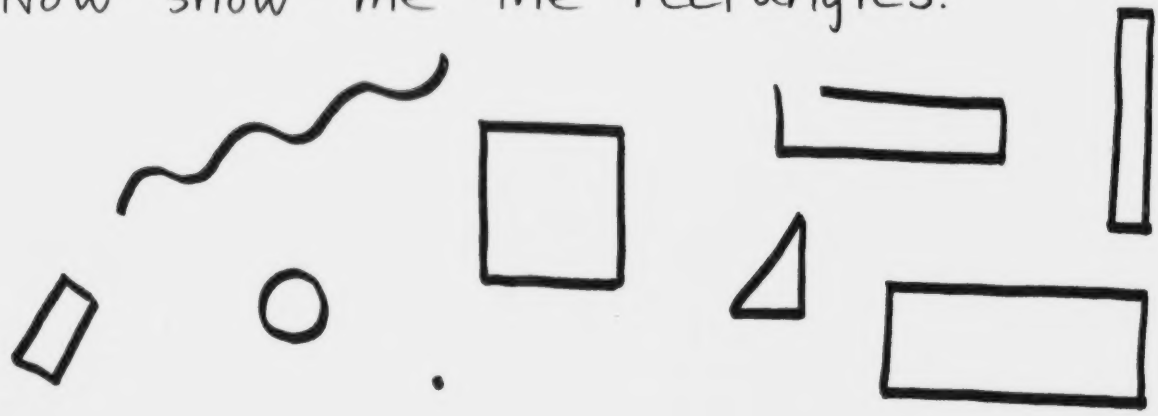


"this one is not a rectangle either."

"Is this one a rectangle?"

(No... it needs 4 sides.)

"Now show me the rectangles."



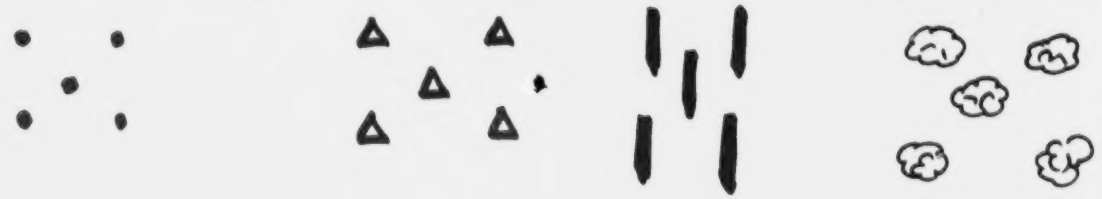
"Show me some that aren't rectangles."

FIVE

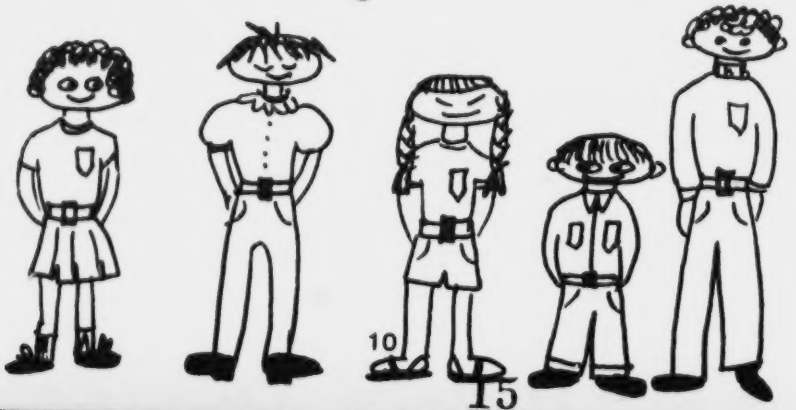
• • • • •

Present 5 in lots of different arrangements

Not always the same arrangement!



"5 children have 5 heads. These also have 5 belts. But are there 5 pockets? Are there exactly 5 shoes?"



Sets of 5 things not exactly ALIKE

5 things to write with

5 things to eat

5 things with wheels

5 tools

5 things to wear

5 things that are red

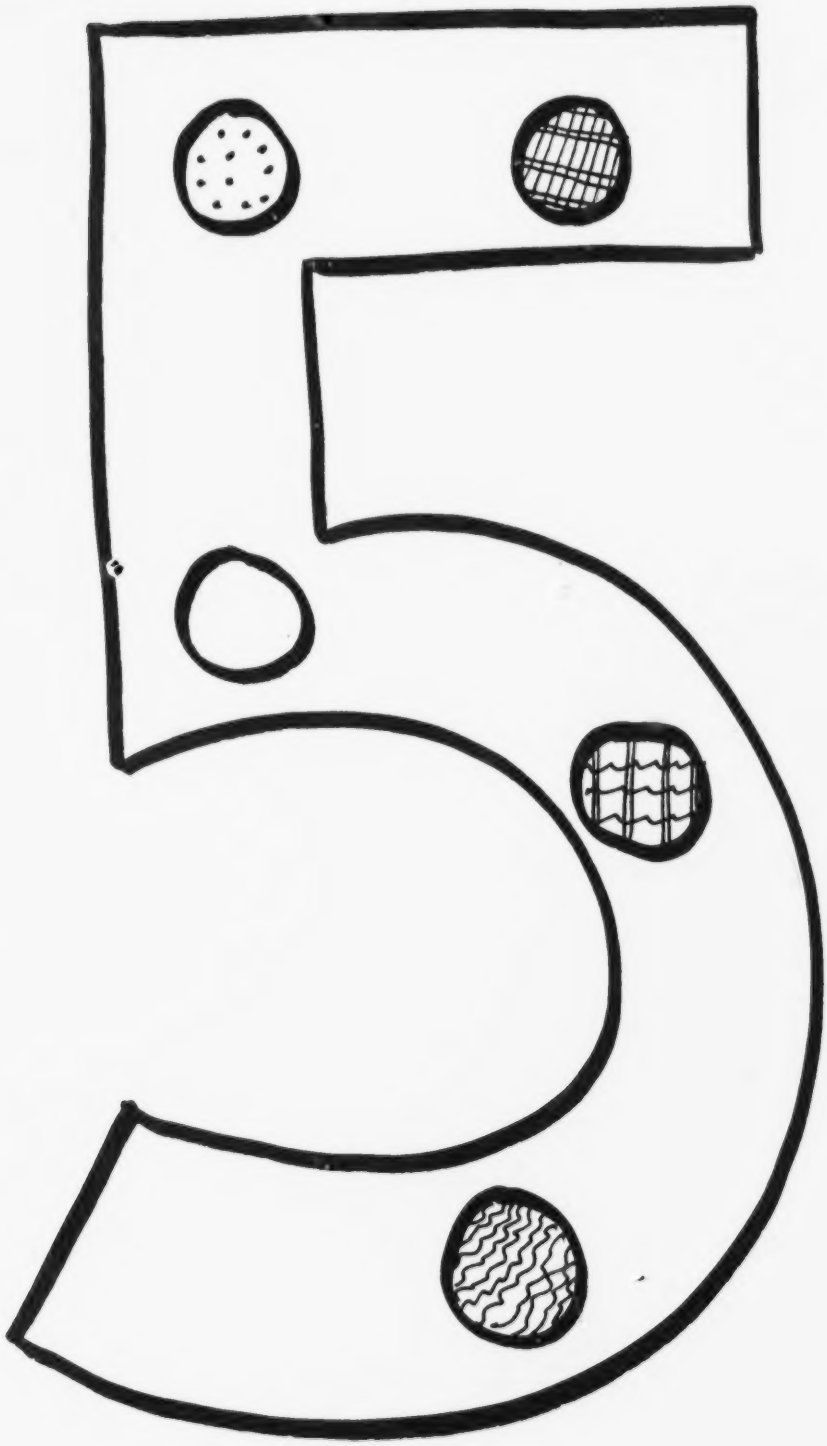
Add to the list . . .

5 things _____

5 things _____

5 things _____

5 things _____

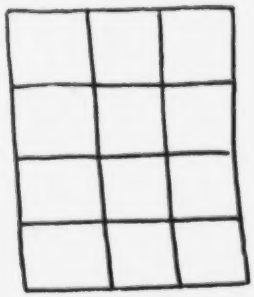


C-6

SOME MODELS of MULTIPLICATION

ARRAYS -- rectangular arrangement

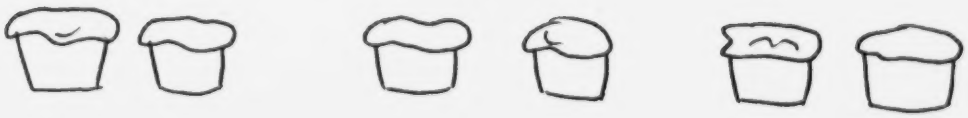
Mom puts carpet squares on the bathroom floor. She uses 3 across and 4 going the other way. (To illustrate, lay construction paper squares on the floor.

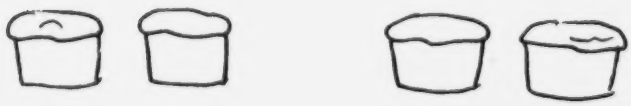


$3 \times 4 = 12$

REPEATED ADDITION -- no. of sets with same no. objects in each set

Kashonda eats 2 cupcakes a day for 5 days.





$2+2+2+2+2=10$

or

$5 \times 2 = 10$

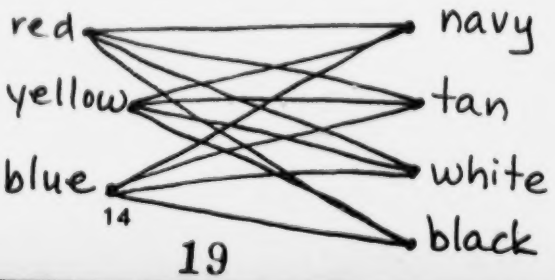
Another Model of MULTIPLICATION

CARTESIAN PRODUCTS (COMBINATIONS) -- Each member of one set is paired with each member of a second set

Kenny has 3 shirts -- red, blue, and yellow. He has 4 pairs of pants :- navy, tan, white. and black. How many different outfits can he make?

He could wear

There are 12 outfits. The red shirt is paired with each pants. The blue shirt is paired with each and the yellow shirt is paired with each pants.



more CARTESIAN PRODUCTS

2 kinds of cones

4 kinds of ice cream

8 different single dip cones that might be arranged!

Importance of CONCEPT DEVELOPMENT

- Enhances thinking ability.
  Avoids dependence.

- Adds meaning to operations, relationships, measurement.

- Teaching for meaning is efficent!
  Time spent on concept development results in:
  - greater retention
  - greater transfer
  - increased problem solving ability
  - increased achievement test scores
  - better computation

ENHANCING CONCEPT DEVELOPMENT

- Work with examples and nonexamples
- Children talk about concepts:
  - give examples
  - explain
- Work with objects and manipulatives
- Work with pictorial representations
- Work with games & puzzles
- Draw or create representations

DEVELOPING MULTIPLICATION CONCEPTS

C-12a

Examples and Nonexamples of Multiplication

(Use the problems previously suggested or similar ones. Where possible, use real objects or patterns.)

For the repeated addition model, use these problems. Have the children work in small groups to arrange the objects and count to find the answers. Help the record their work.

Problem

three children each had 2 pencils. How many pencils did they have in all?

Recording of Work

$2 + 2 + 2 = 6$

And here's a shorter way:

$3 \times 2 = 6$

(3 groups of 2)

Have children hold pencils. Try other combinations

Problem

Cory, Billy, Tim, and Greg each ate 3 hot dogs. How many hot dogs did they eat in all?

Recording of Work

$3 + 3 + 3 + 3 = 12$ or

$4 \times 3 = 12$

children arrange hot dog cutouts. (pattern C-13)

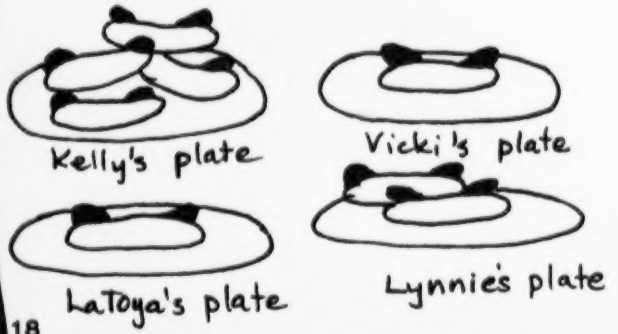
Now use a nonexample . . .

Kelly ate 4 hotdogs. Vicki and LaToya each ate just one. Lynnie ate 2 hotdogs.

Recording of Work

$4 + 1 + 1 + 2 = 8$

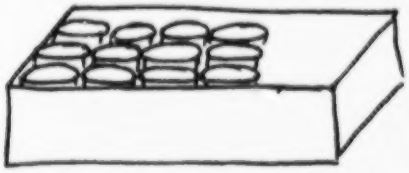
there's no shortcut here. the groups aren't equal!



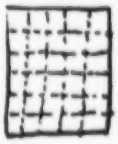
Some examples of the arrays or rectangular arrangement follow.

Problem
Keesha put some empty soft drink cans in a box. She packed them neatly in the corner like this (show). How many cans?
Recording of Work
$4 \times 3 = 12$

Use a real box and clean empty cans. Next let children create different rectangular arrangements in the box --- perhaps 3x3, 4x5, 6x2. Have them count the number of cans and record their answer.



Problem
Mrs. Washington bought some stamps. They were in a rectangular shape 5 across & 6 down. How many stamps?
Recording of Work
$5 \times 6 = 30$



Here the children might draw what they think the stamps looked like. They should discuss various strategies for getting the answer --- counting by 1's, by 5's, etc.

We already used two examples of Cartesian products or combination-type multiplication. Below is one more:

Problem
The Brown family has some cups and saucers. They have cups in red, blue, green, & yellow. They have white, green, and tan saucers. How many different ways can they arrange the cups & saucers.

Use the paper cutouts from pattern C-14. Children could color these. Arrange all the possible combinations. Of course, there will be 4x3 or 12 combinations

etc ...

Children Explaining What They Are Doing. Children work with manipulatives while they show and tell each other what they are doing. Teachers should also help children record their work, but the focus should be on the process of solving the problems not on the answers.

Problem

Put 2 eggs in each basket. How many eggs in all? (Use various numbers of margarine cup baskets & large lima beans

Recording Work

$3 \times 2 = 6$, $7 \times 2 = 14$, etc

Have children set up the situation and find the answer, then tell about their work.

Problem

Arrange the children in rows. Find out how many children there are. For example, use 4 rows with 3 children in each.

Recording Work

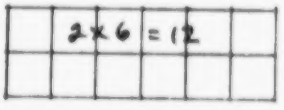
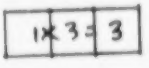
$4 \times 3 = 12$

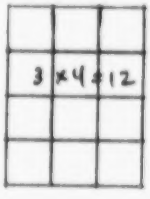
Let some children direct others to set up various arrangements. Also use cutouts on a flannel board.

Handling Manipulatives. Along with "talking mathematics," children need objects to arrange, color, cut out, and work with in other ways. Let's look at some more situations that help children learn about the nature of multiplication as they handle manipulative materials.

Rectangular Arrangements

Draw and cut out rectangles of various sizes. Lable each. Find the total number of squares.

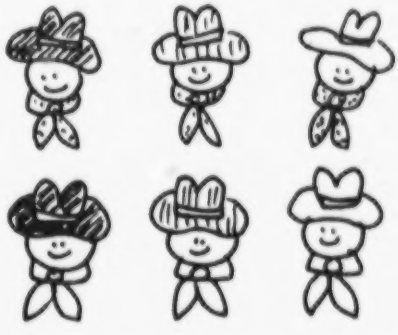
Use C-15 grid paper.





Hats and Scarves

Color the hats and scarves and face cutouts (Pattern C-16). Cut out the patterns. Children arrange the cutouts to show and solve problems. For example:

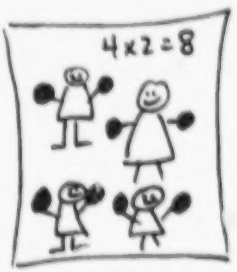
Green, red, and brown hats
Yellow & white scarves
How many different "outfits"?



Pictorial Representations. Children can draw their own representations and choose from representations drawn by adults to show various multiplication situations.

Problem

4 children. Each has 2 balls. How many balls. Draw a picture. Show and write the answer.



Problem

Which card shows:

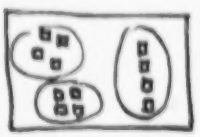
3 groups of 4

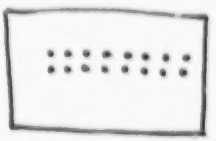
2 x 8

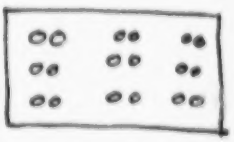
7 groups of 3

Write the answers.

(Pattern C-16 has examples)





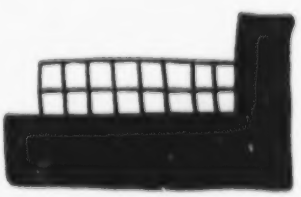


Array Maker

Arrange the "L" shape over grid paper to make arrays like these

2 x 8	9 x 1
3 x 4	4 x 6

Count the squares & get the answers. (Pattern C-17)



"2 x 8 = 16"

Child holds the "L" shape over grid paper so only 2 x 8 shows.

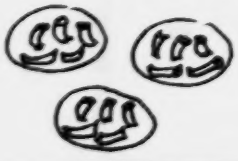
Using Related Games and Puzzles. Children can work with multiplication games and puzzles independently or in small groups.

Bugs 'n' Rugs

Children use task cards and arrange cutout bugs on rugs. They record their results. If task cards are laminated, they can write on them with crayons, and rub off the crayon for reuse. (Pattern C-18)

Macaroni Dinners

children arrange macaroni pieces on small paper plates according to task cards. They find the total number of pieces and check with the number on the back of each task card

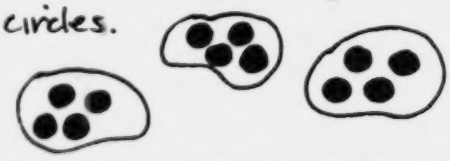


3 x 5 front

3×5=15 back

Lots of Circles

Children work with plastic counters -- perhaps gallon milk jug lids. They spin 2 factors on a spinner (pattern C-19), then arrange the lids within yarn circles.



Puzzle

Joe started with 5¢ and doubled his money for 5 days. Ketrick started with 1¢ and tripled his money each day for 5 days. Who had more money at the end? Use play money or real coins. Let children work out the puzzle. The chart shows results -- Ketrick had 1¢ more on the 5th day.

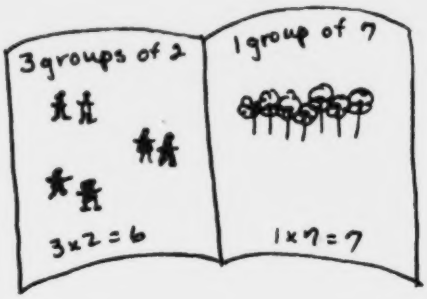
Day	Joe	Ketrick
1	5¢	1¢
2	10¢	3¢
3	20¢	9¢
4	40¢	27¢
5	80¢	81¢

22

Drawing and Creating Representations. As children create their own representations of mathematical situations, each is actively involved in concept development. Teachers and assistants should be sure to appreciate differences in approaches to making representations -- most concepts can be correctly illustrated in many ways.

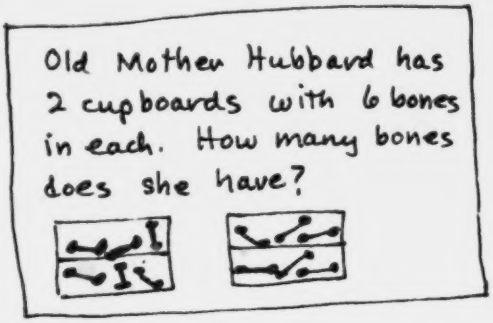
Cut and Paste Groups

Children make booklets to illustrate several multiplication facts. They cut pictures from catalogs, newspapers, or magazines to illustrate facts. They label each picture with a recording of the fact and answer.



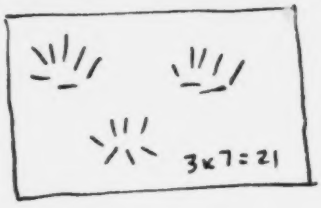
Draw and Solve

Children write (or dictate stories for adults to record), then draw pictures to illustrate the stories. A group may work to solve a story problem, or children may work individually.



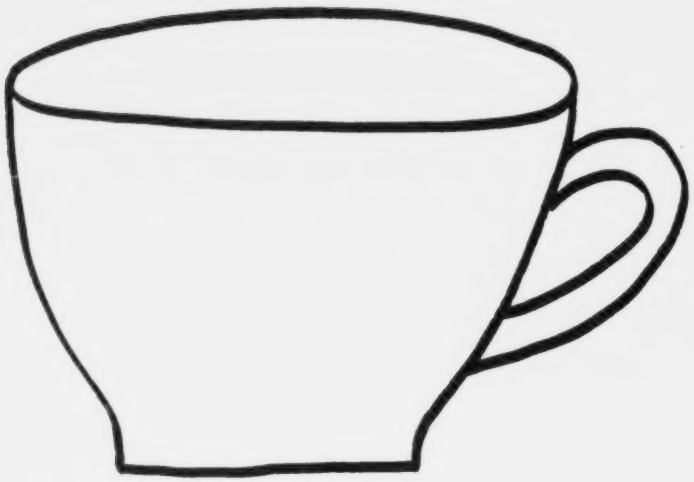
Stick Facts

Children glue papers of toothpicks to illustrate multiplication facts. They label each illustration.

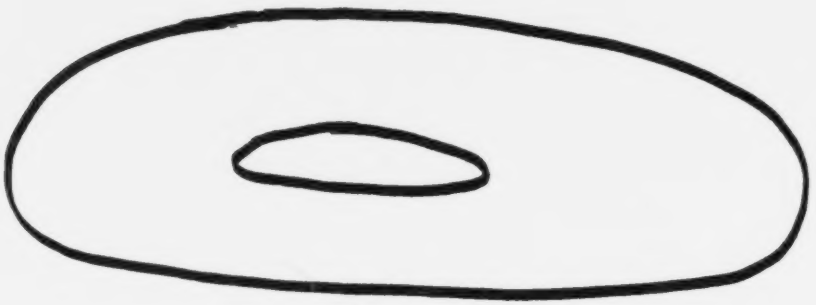


Patterns

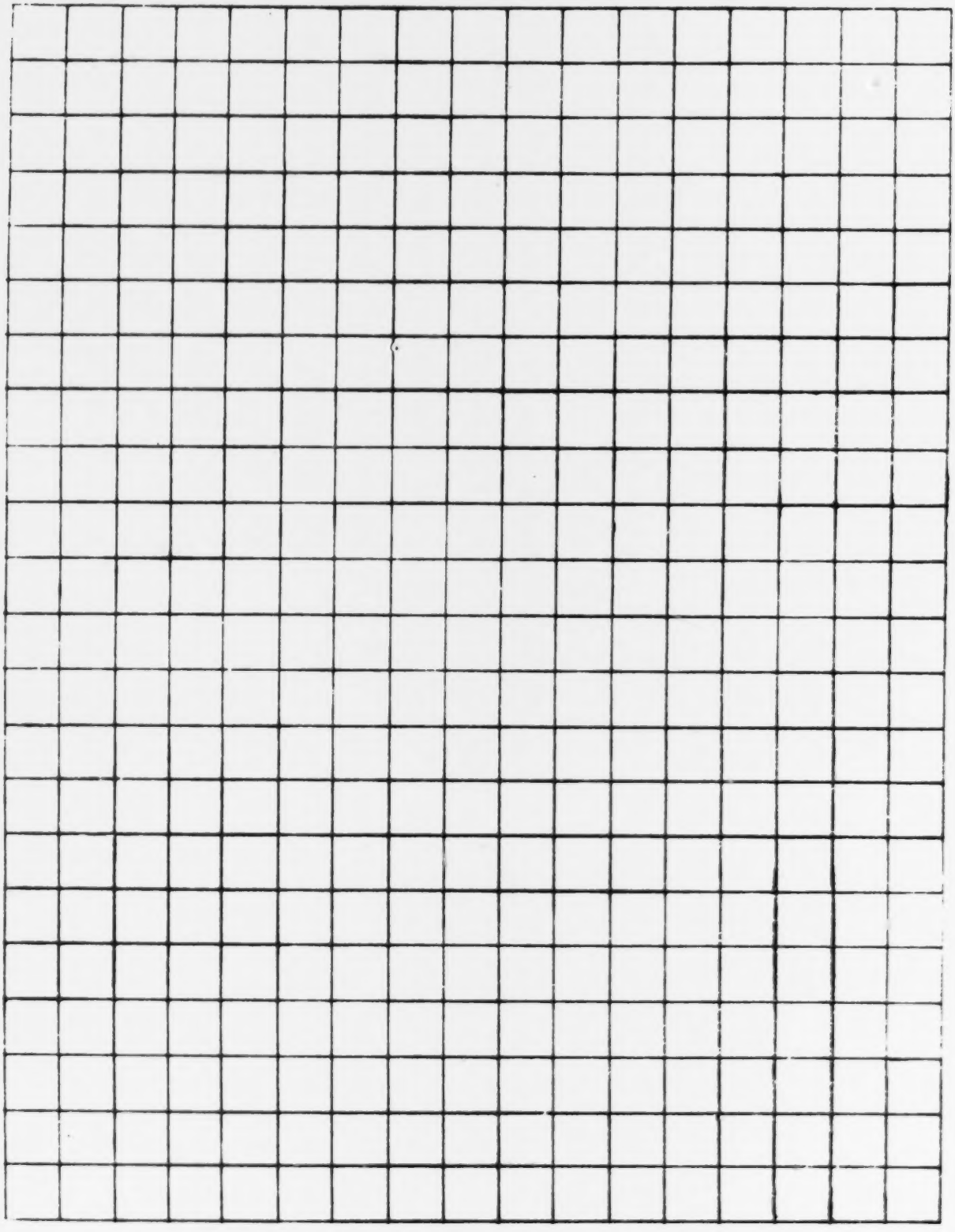
C-13
Hot dogs

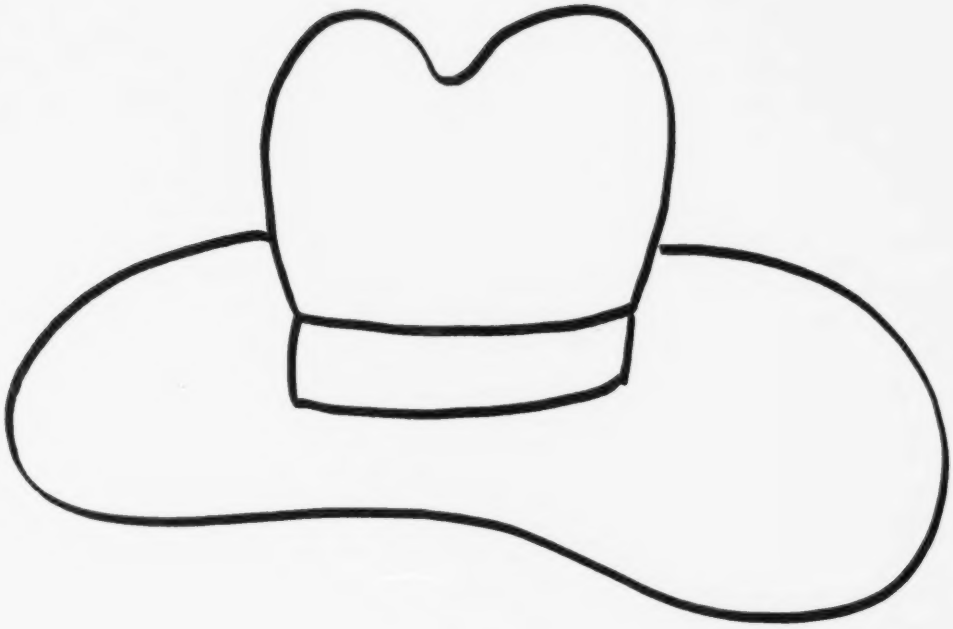


C-14
Cups and saucers



One-Centimetre Grid

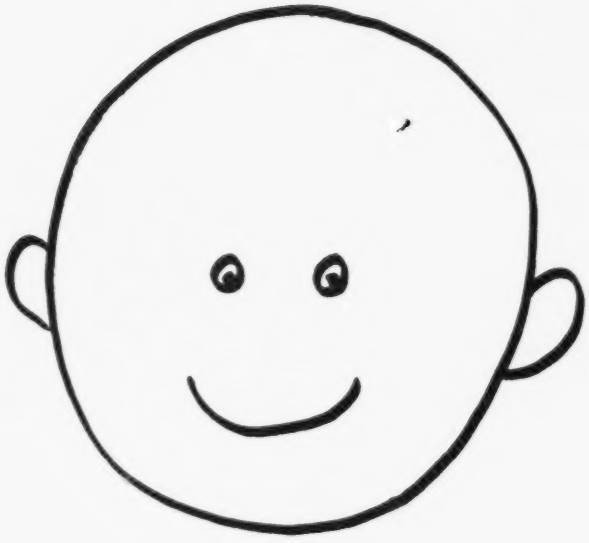


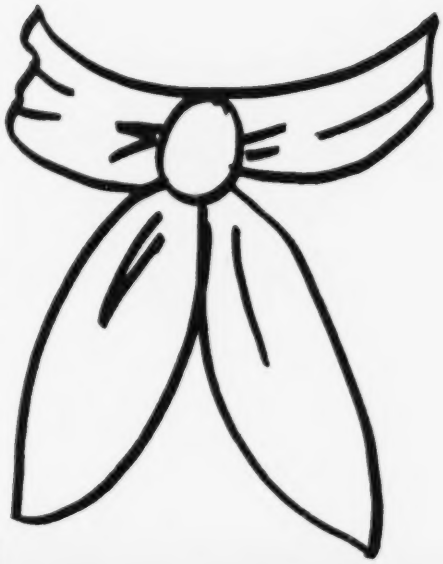


Patterns

Cowboys, hats and scarves

C-16

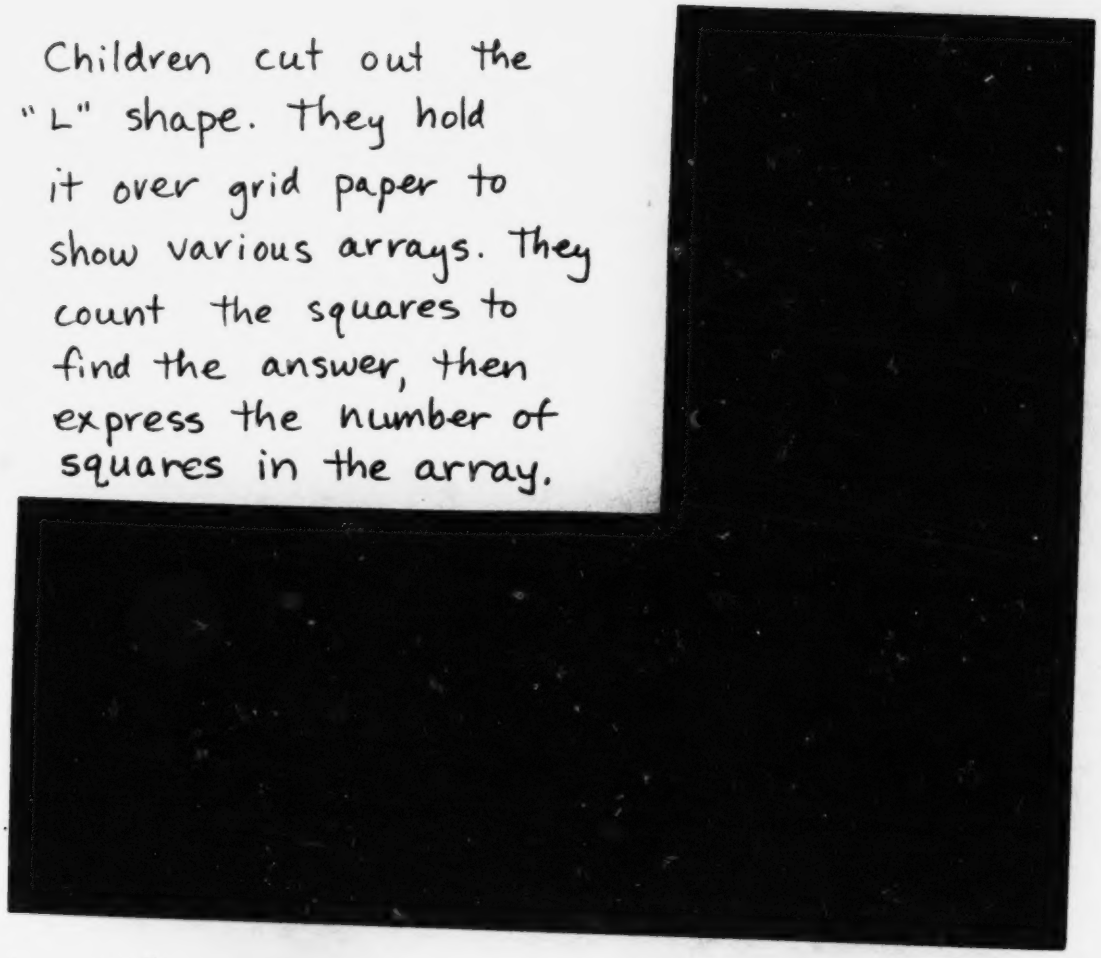




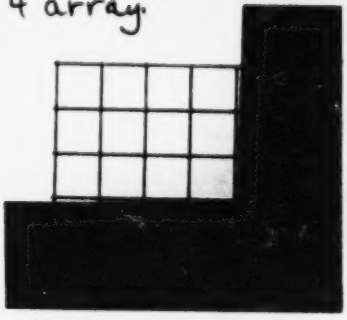
Pattern -- ARRAY MAKER

C-17

Children cut out the "L" shape. They hold it over grid paper to show various arrays. They count the squares to find the answer, then express the number of squares in the array.



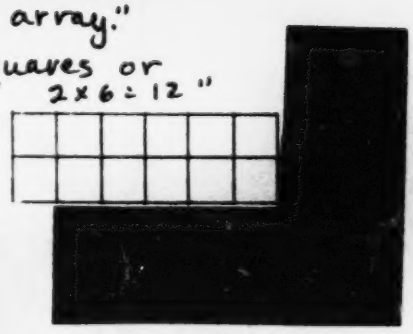
"3 by 4 array."

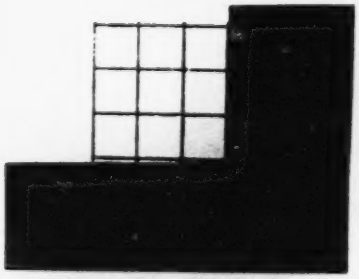


"12 squares in all. 3x4 = 12."

"2x6 array."

"12 squares or 2x6 = 12"



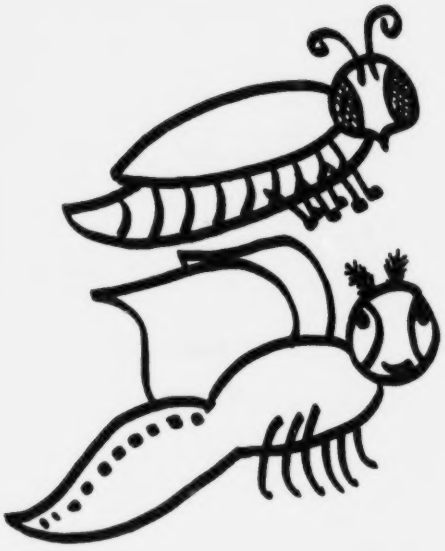


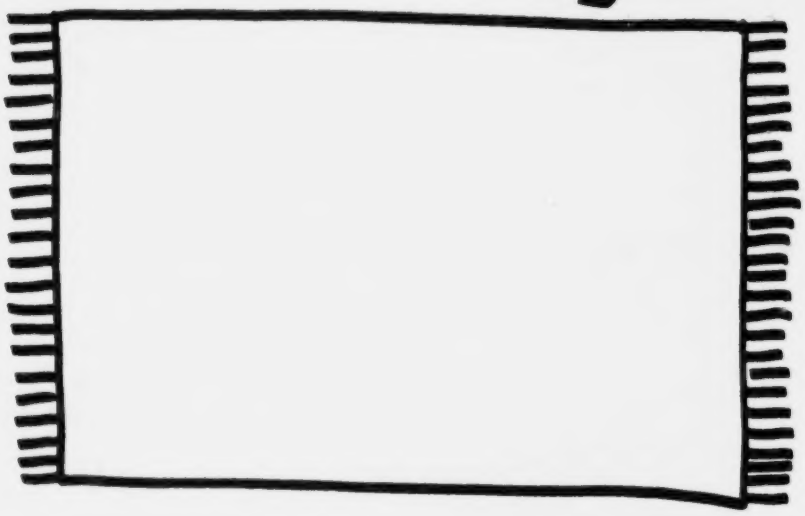
"3x3 array"

"9 squares or 3x3=9."

Bugs 'n' Rugs

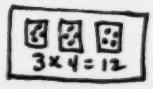
Pattern



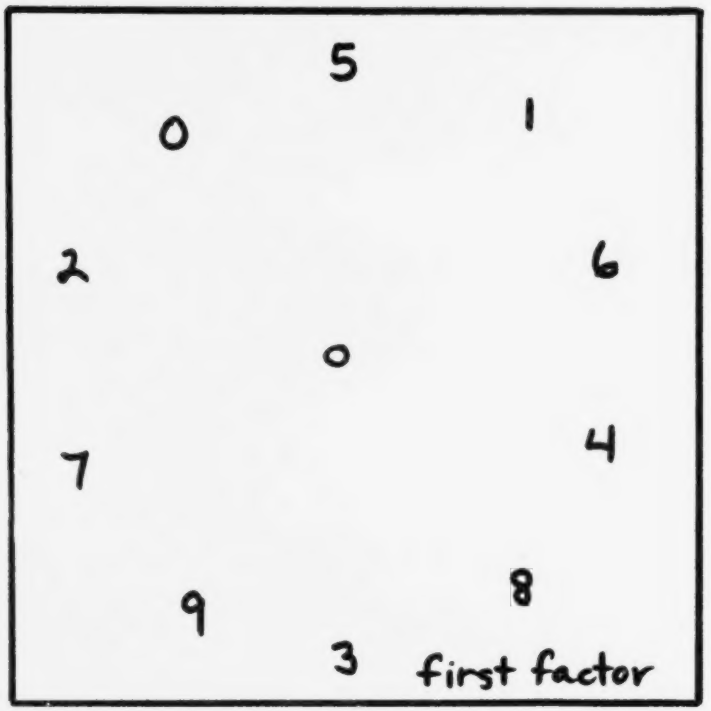


Duplicate 40 (or more bugs) and 10 rugs. The children can color these. Make task cards such as those below. Have children arrange bugs on the rugs and count to find the answers. They can record the results.

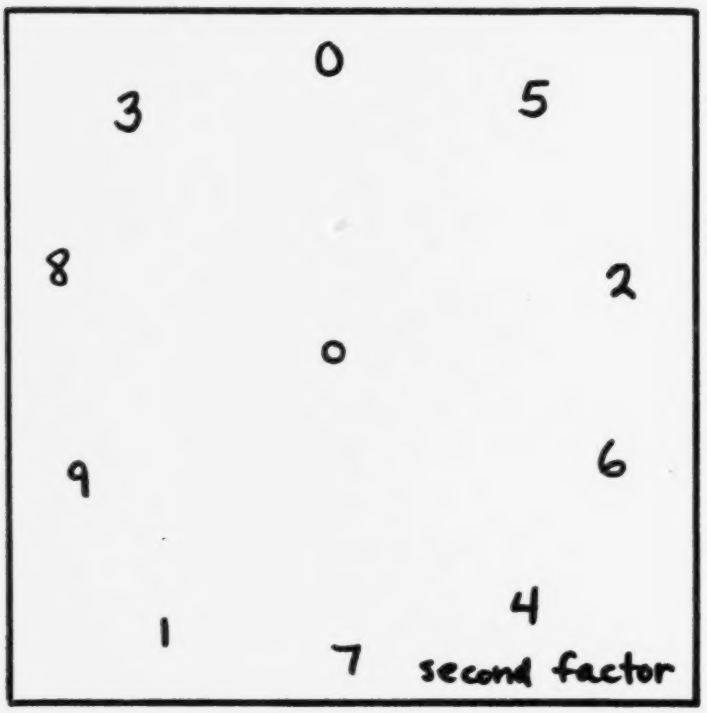
4 rugs. 2 bugs on each rug. How many bugs in all? ☐ x ☐ = ☐	3 rugs. 5 bugs on each rug. How many bugs in all? ☐ x ☐ = ☐
7 rugs 4 bugs on each rug. How many bugs in all? ☐ x ☐ = ☐	9 bugs in all. 3 rugs with the same number of bugs on each rug? How many bugs on each rug?

Code backs of cards with answers: 

A Simple Spinner



Attach a large safety pin or paper clip to the spinner's center with a brad. Make sure the spinners work freely.



Children spin 2 factors & tell the product.

C-19

SKILL DEVELOPMENT

SKILL DEVELOPMENT

In this workshop session, participants will receive information on the nature of mathematics skills for young children and how skills and concepts development are related. Participants will learn about principles for skill development and keeping skills viable over a period of time. They will explore materials and techniques for development of various skills. Finally, participants will share more ideas for skill development. This session could be profitably followed by a **Make It — Take It** session in which participants create materials for use in skill development in their own classroom.

The workshop leader should:

1. Discuss skills, give examples of mathematics skills for young children, and encourage participants to discuss skills expected of young children.
2. Relate skill and concept development through examples.
3. Present principles for skill development.
4. Briefly explain and demonstrate materials and techniques for skill development, then allow time for browsing by participants.
5. Encourage participants to share more ideas for skill development.
6. Prepare materials prior to the workshop session:

   Transparencies S-1 through S-8.

   Handout sheets S-10; Transparencies S-5 and S-6 might also be made into handout sheets.

   Materials Needed for Activities:

   Use S-11 and Patterns S-12 through S-18 to make up materials. As stated in the concept development workshop, if the leader cannot make the materials, a classroom teacher and assistant may be willing to do so, and then keep the materials for future use.

   Also secure a chalkboard and overhead projector.

1. What are skills? (Transparency S-1) Skills are mental or psychomotor activities that children must be able to perform quickly and efficiently. Saying answers to problems on flashcards or placing a ruler on an object and reading length of the object in centimeters are skills. When children count coins to equal a quarter or dollar, they demonstrate skills in dealing with money. Children use their skills as they read a calendar or clock and when they write numerals. As children demonstrate their problem-solving abilities, they also show that they have acquired both computational and reasoning skills.

   Some examples of mathematical skills that should be acquired by most primary grade children follow:

   Counting. Children count forward and backward, usually first to 10, then to 100. As children study larger numbers, they can be taught to count forward and backward from a given number in exercises such as "Let's start at 203 and count forward for a while . . . 203, 204, 205, 206, 207, 208, 209, 210, 211 . . ." After they count skillfully with ones, children should have practice in counting by tens, fives, twos, and threes. Practice with tens helps with readiness for multiplication and division.

   Reading and writing numerals. After children have some idea of what the numbers mean they are studying, they practice on reading and writing numerals. By the third grade, children read numerals up through thousands. Writing numerals has both psychomotor and intellectual aspects. Children must control their hands to form numerals and must also have understandings of place value to know what numerals greater than ten mean.

   Basic facts. One of the main tasks for children in the primary grades is the association of basic addition, substraction, multiplication, and division facts and their answers. Most children, with proper practice, can learn to say and

write the answers to basic facts.

Performing Written Computations. Beyond basic facts, children learn to deal with addition and substraction problems where muitidigit numerals are involved. Further, regrouping in both addition and substraction are introduced and reinforced in the primary grades. Using pencil and paper, children should perform written computations with reasonable speed and accuracy.

Measurement Tasks. Children acquire competency in handling measurement devises such as rulers, clocks, and calendars. They should be able to compare relationships among coins up to a dollar, identify coins, state or show equivalent combinations of coins, make change, and add up the values of given coins. Further, primary grade children learn to name, draw, and match basic geometric shapes such as rectangles, squares, triangles, and circles.

Problem Solving. Primary grades children should apply their computational skills in problem solving situations. Problems may be presented in oral or written form. Children should deal with some obvious problems (additional situations as the children study addition) and also some not-so-obvious problems that are thought-provoking. (These training materials include a workshop on problem solving.)

2. It's not either skills or concepts! Skills and concept development are closely related and should be inseparable at any grade level. (Transparency C-3) Skills and concepts are complementary and should be worked on together. Without skills, concepts are useless. Ideas need to be put to work! (Transparency C-4) Without concepts, skills don't mean much; skills become silly things that are done without understanding. Although this workshop focuses on skills, it has been preceeded by a workshop on concepts. It will be followed by a workshop on skills and concepts "hand-in-hand" (could show Transparency C-3 again). In the classroom the two areas should be so constantly related that they're almost inseparable.

Transparency S-5 (or make this into a handout sheet) shows some activities that develop concepts and skills concerning money. The activities would be most appropriate for small groups of children led by a teacher or assistant. The activities use real coins, pictures of coins, and coin rubbings done by the children. The activities involve different modes of presenting information and responding-verbalizations, drawings, handling manipulatives, writing, and pictures. In working with the activities, adults should be realistic about attaching prices to objects. While it might be tempting to show a picture of a large candy bar labeled 5¢, children would be hard-pressed to find such a value in a store. (Ask participants to add activities that would strengthen both concepts of skills about pennies, nickles, and dimes.)

For another example of the close relationship between concepts and skills, let's look at an introduction to numbers in the hundreds. Prior to work with hundreds, students would be able to write most or all of the numerals 0-99 and have dealt with place value in tens and ones. Transparency S-6 shows several activities for concept and skill development dealing with numbers in the hundreds. What other activities could be used to extend and expand the children's understandings and skills?

3. Some principles for skill development. (Transparency S-7) Guidelines for skill development in mathematics may also apply to other curricular areas. Some principles to guide educators in thinking about skill development are:

Provide much practice with a variety of materials. The math skills young children must master seem relatively easy to adults, but adults have had years and years of practice! Adults must appreciate the need for lots of practice. To prevent boredom and to "reach" children who learn in different ways, a wide variety of materials must be provided. Adults can vary the appearance of materials — children might practice the same skill of mastering basic addition facts with the "Addition Clown," and "Horses and Saddles," and with the "Addition Family Cards" (Transparency S-8).

Some children will respond best to verbalizations, songs, and other aural stimuli. Others will learn more with tactile approaches in which they handle materials with different shapes and textures. Still others react best to visual stimuli — lots of interesting things to look at.

Spread out practice periods over a period of time. Few children master skills in a short time period. Therefore adults should provide frequent periods where children can practice skills. Practice periods need not be long — working with a skill 5 minutes a day for 10 days is usually more effective than working on the skill just once for 50 minutes.

Keep skills alive over a period of time. Forgetting is a natural and very real aspect of learning. Factual knowledge is quickly lost without regular practice and use. Review skills both in isolation (work on one skill at a time) and develop a day's work that involves several skills in an intriguing problem-solving situation.

Drill on basic facts is important but should not be the only focus of the primary grades mathematics program. Some special guidelines, based on research, apply to drill. (Transparency S-9)

- Children should attempt to memorize facts only after, but soon after, understanding or concept development has been attained. Otherwise facts being memorized don't mean much.
- Children should participate in drill exercises with the intent to memorize. Remembering should be emphasized. Explanations should have preceeded drill time or may follow it.
- Drill sessions should be short, but should be given almost every day until skills are mastered. Drill sessions can become a routine part of classroom procedures.
- Children should try to memorize only a few facts at a time, but should constantly review previously memorized facts.
- After initial drill sessions, children will vary in their rate of memorization. Provisions must be made for both "faster" and "slower" children.
- Children must build confidence in their ability to memorize. Praise and rewards (both tangible and intrinsic) should be given. Individual records of progress are often motivating.
- Drill activities should be varied, interesting, and conducted with enthusiasm. This provides a challenge to adults who must plan and lead many different drill sessions.
- Children need ways to quickly assess the correctness of their answers on drills. Adults can listen to answers, as can classmates who know the answers or who have an answer key. Many self-checking devices can be used. Children can separate known and "missed" facts into two piles and "redrill" on facts they missed.
- Where children err, they should repeat the entire problem and answer, not just the answer. This procedure builds associations of problems and answers.
- Where children err, they should repeat the problem in a different modality than previously given. If children are saying answers to each other aloud, they might also write the entire problems they miss. If children are writing answers, they should be encouraged to say the problem and correct answer (even in a soft voice, talking to one's self is reinforcing). Children who miss basic facts on flashcards could trace around the numerals while saying the answers.
- Timed drill exercises are helpful, especially when participants keep track of their own results (such as 10 addition facts correct in 30 seconds) or are fairly evenly matched in ability.

4. Selected materials for skill development. Use participant Handouts S-10 and the notes on the workshop leader's copy, S-11. Briefly overview skill-building activities, then allow time for browsing and examination of materials. Most primary teachers and teaching assistants will be motivated to look at and try out materials. For groups with less motivation, ask

participants to examine the materials and find answers to questions such as these:

- Find something a particular child you're teaching now could profit from using.
- Find an activity you could modify to teach another topic. (Changing a subtraction activity into one for teaching about fractions for example.)
- Find an activity to suit an especially verbal child, or one who learns best from aural or tactile stimulation.

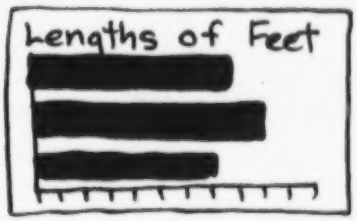
After a period of browsing, see if participants have questions about any activities. Let several participants tell which activities they thought were effective and why. Lead participants to discuss some of the above questions.

5. Sharing more ideas for skill development. Ask participants to choose other skill areas and verbally share ideas that they have used effectively. If possible, someone could take notes for future duplication and distribution of ideas. This workshop could be followed by a session in which participants prepare materials — from the workshop ideas or other sources — to promote skill development in their classrooms.

SKILLS

"five"

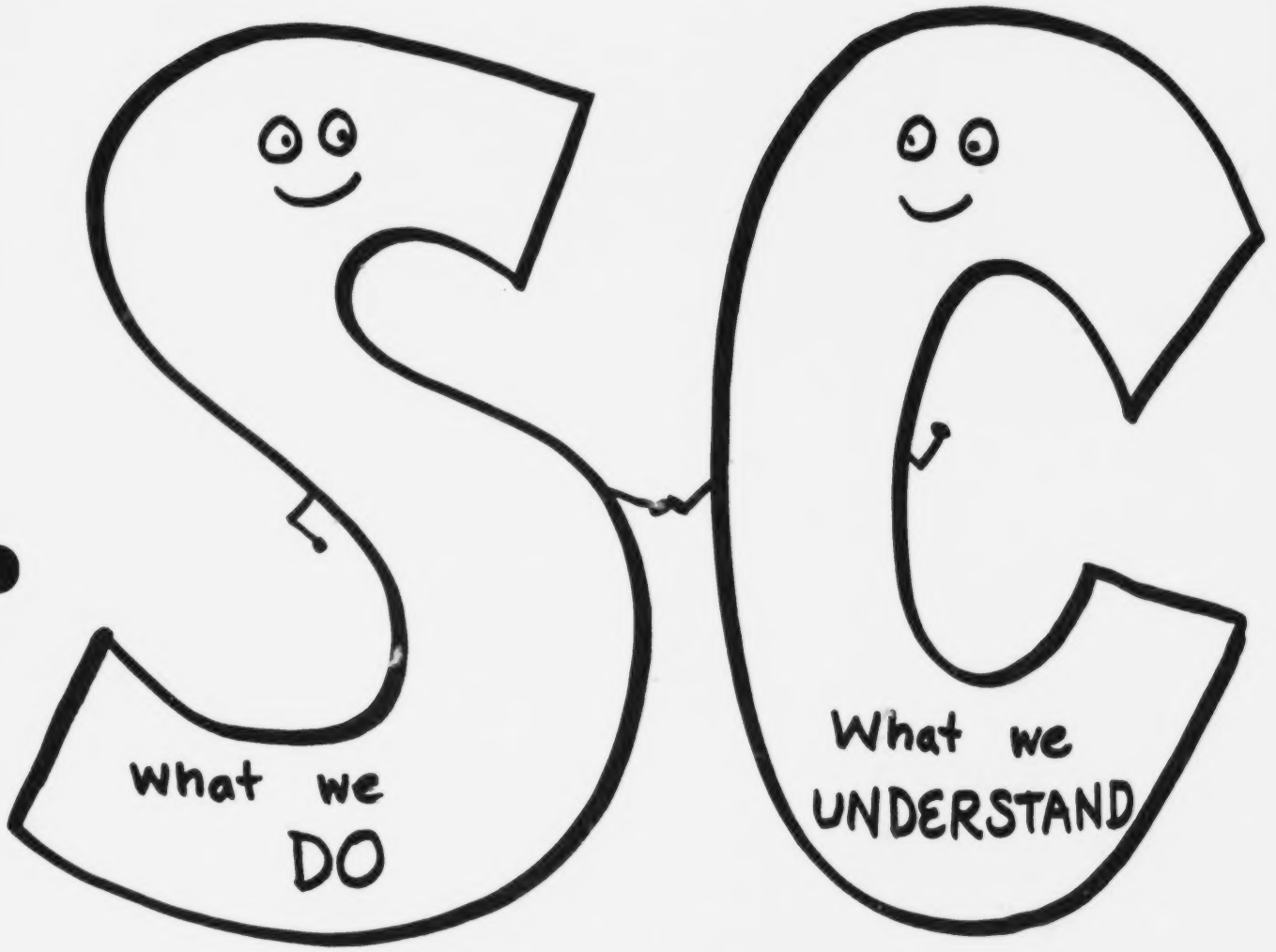
$$\begin{array}{r} 13 \\ -8 \\ \hline \end{array}$$



$$\begin{array}{r} {}^{4}\cancel{5}{}^{9}\cancel{0}{}^{10}0 \\ -\ 198 \\ \hline \end{array}$$

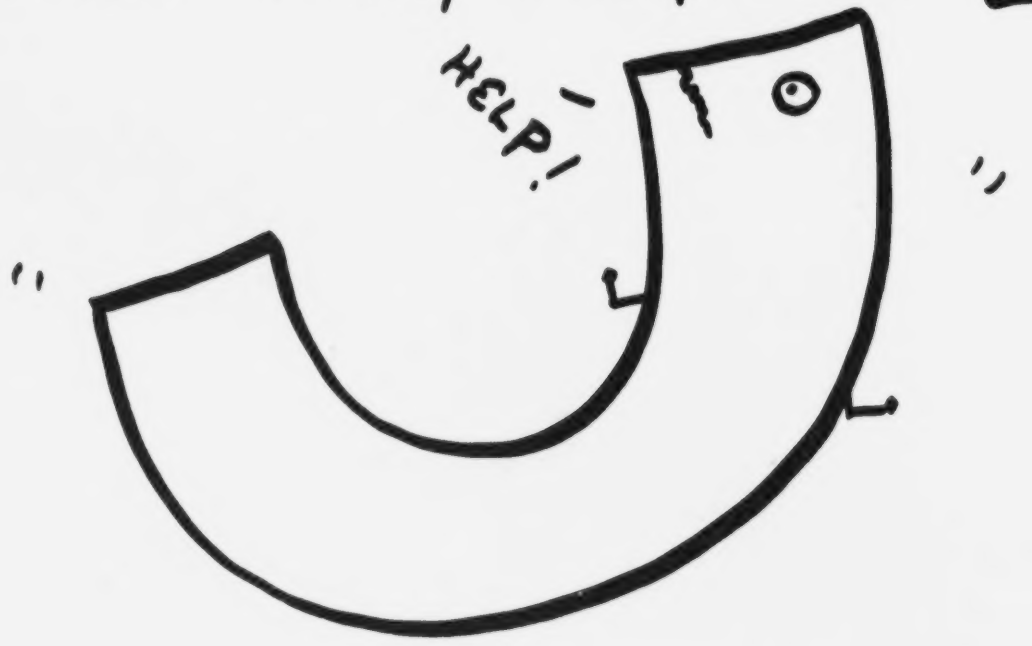
- **SKILLS** -- WHAT WE CAN <u>DO</u> QUICKLY AND EFFICIENTLY
  - Counting
  - Reading & writing numerals
  - Saying or writing answers to basic facts
  - Performing written computations
  - Using measurements
  - Problem Solving

- Remember...

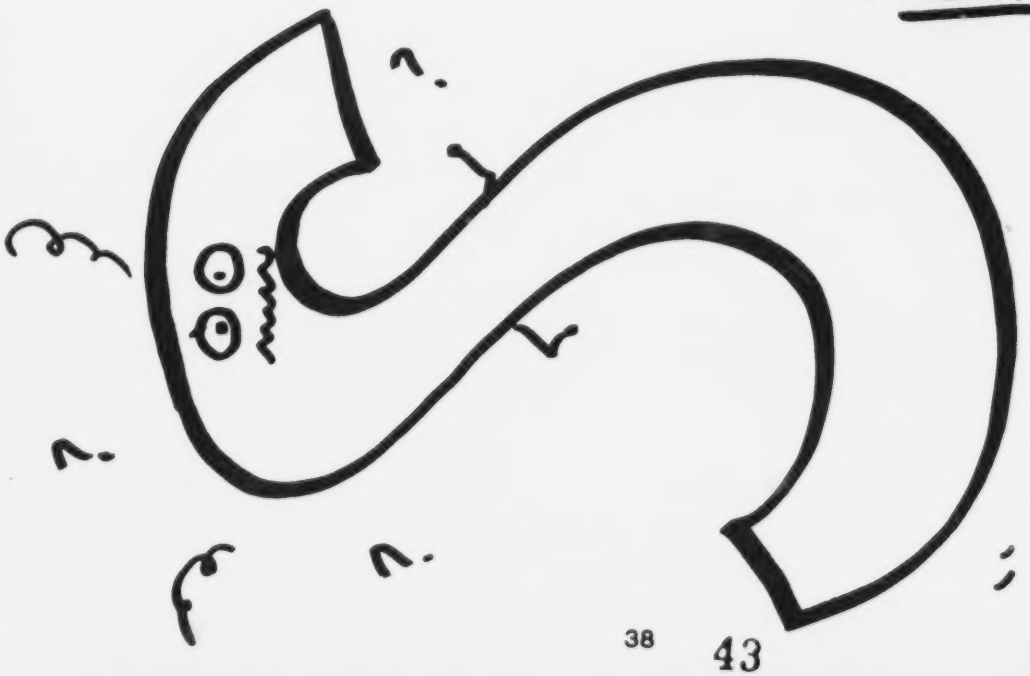


- Skills and Concepts MUST go hand-in-hand!

Without skills, concepts are <u>useless</u>!



Without concepts, skills are <u>meaningless</u>!



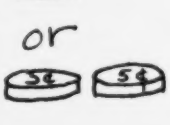
Money -- names & values of penny, nickle, dime

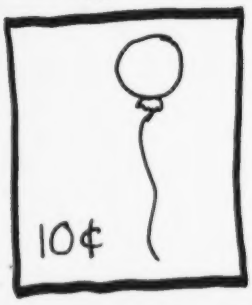
Concepts

- Children talk about and handle coins. Discuss names.
- Rub over coins with crayon. Talk about appearance & worth.
- Work with real coins & exchanges. 1 nickle = 5 pennies.
- Use small real objects. See what coins will buy. Place coins by objects.

Skills

- Identify coins as adult asks "Show me the dime" etc.
- Name rubbings. Write values.
- Lay coins on cards or rubbings.
- Match coins & pictures.





Introducing Hundreds

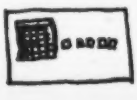
Concepts

- Introduce hundreds with discussion. What follows 99? 105? 238? 605?
- Discuss where nos. larger than 99 are used
- Show 56, 98 with counters. Bundle 10 tens to = 100. Show that 1 ten = 10 ones & 10 tens = 100. We use groups of ten always.
- Children use counters to show number values

Skills

- Count aloud in series such as 99 - 110, 205 - 222, 398 - 415, etc.
- Dictate nos. in hundreds. Children write on board & repeat names.
- Say and write names of numbe shown by count

- Match num numeral card

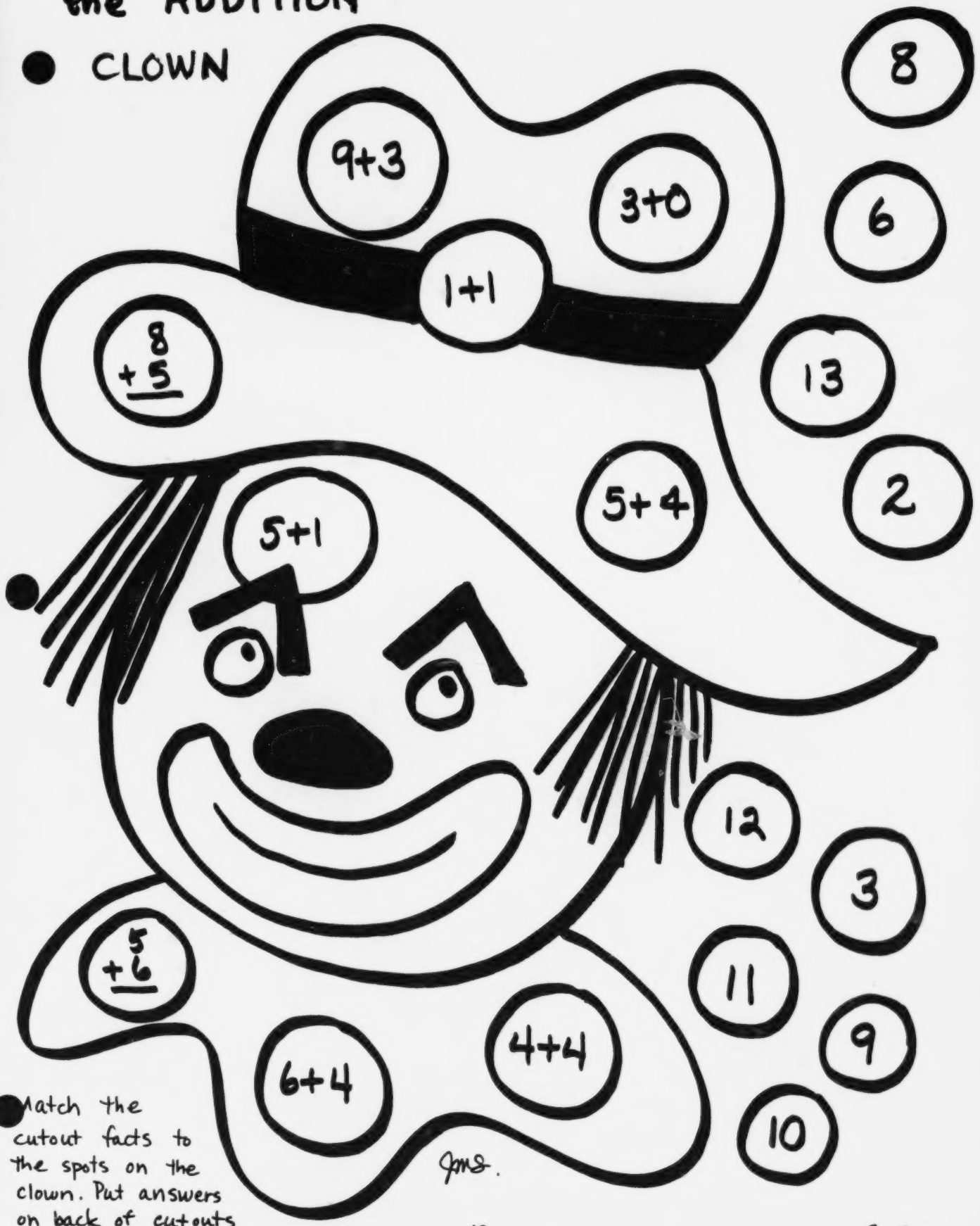


Principles for SKILL DEVELOPMENT

- Provide practice with a variety of materials.
- Spread out practice over a period of time.
- Keep skills alive.
- Drill on basic facts.

the ADDITION

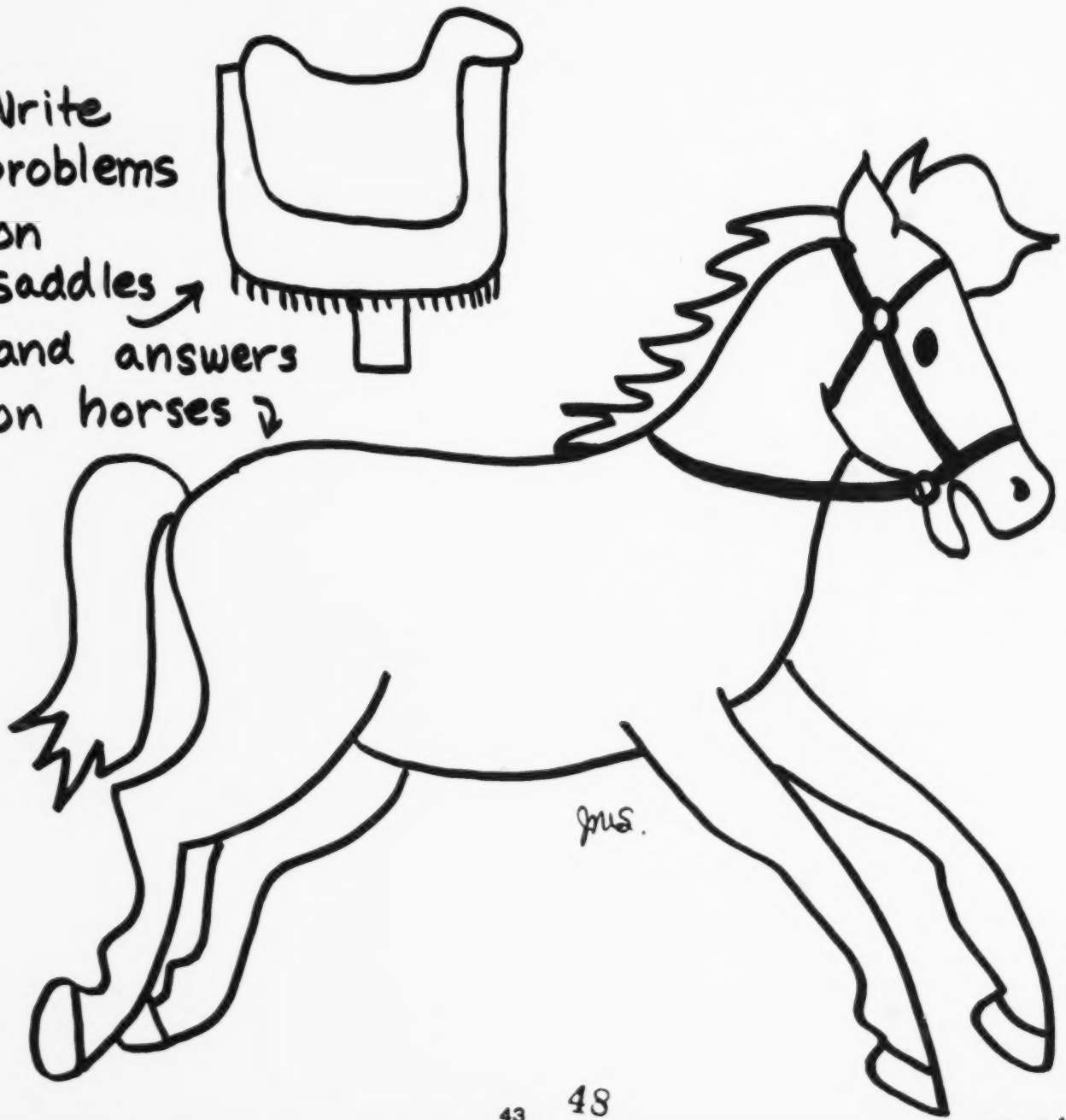
CLOWN



Match the cutout facts to the spots on the clown. Put answers on back of cutouts for self-checking.

Horses and Saddles -- children "saddle up" each horse. Horses could then be lined up from smallest to large answers.

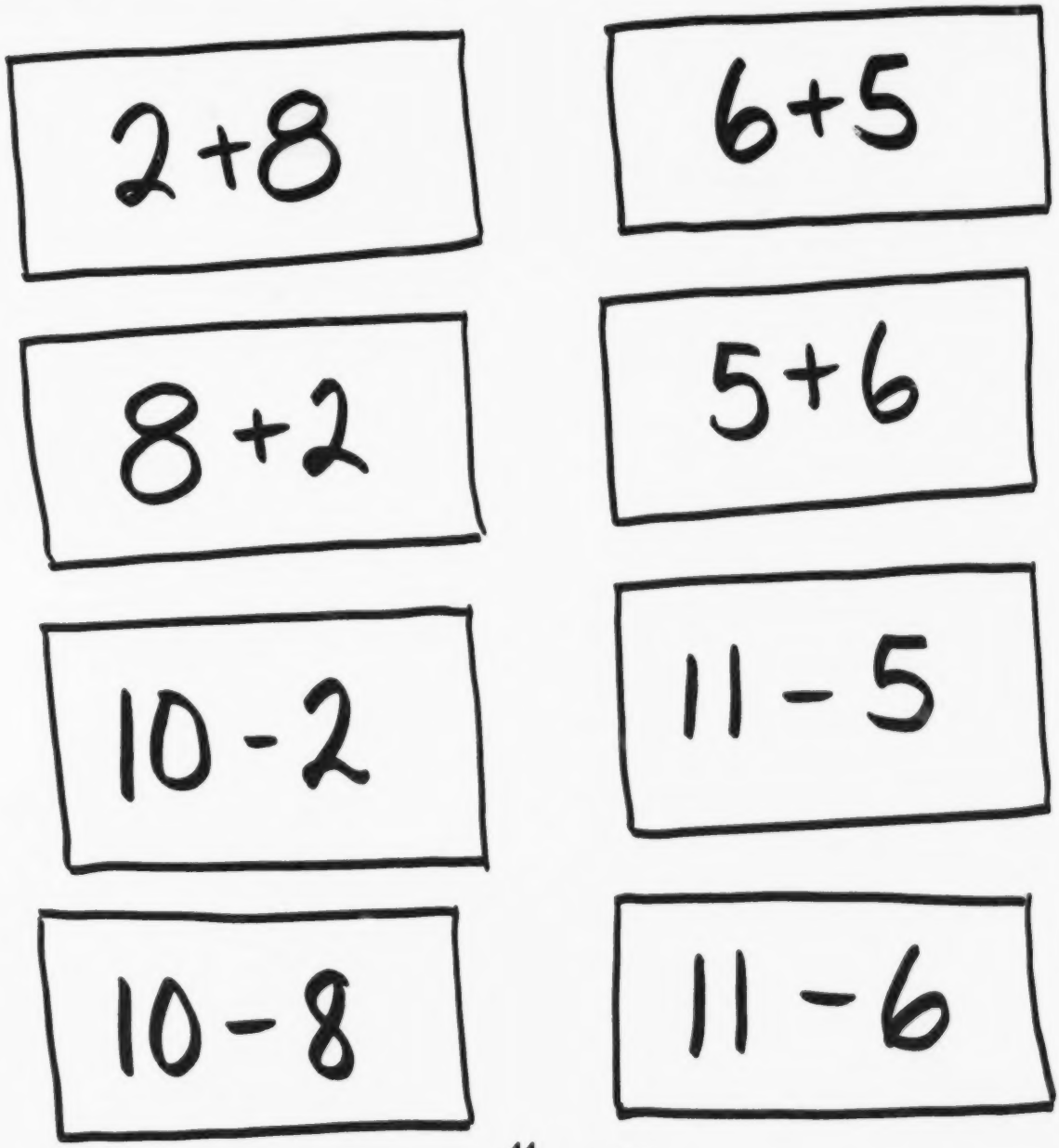
Write problems on saddles and answers on horses



S-8b

ADDition Family CARDS

children group cards from the same family together. Related facts are on the backs. Children say all the answers. Example:



for DRILL, remember...

- Memorize soon after concept development.
- Use drill with intent to memorize. Explain before & after.
- Keep sessions short.
- Try to memorize a few facts at a time. Review.
- Provide for individual rates.
- Build pupil confidence.
- Vary drill routines.
- Let children see if they're correct.

more DRILL guidelines...

- When children err...

  Repeat entire problem.

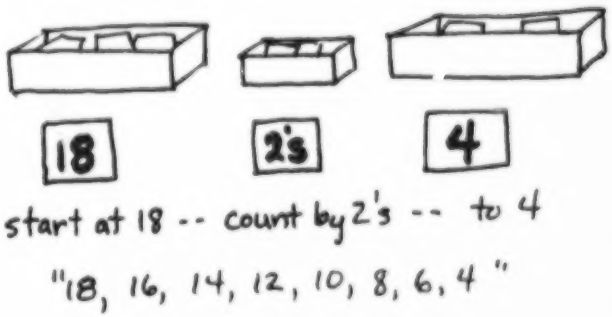
  Repeat problem in different modality.

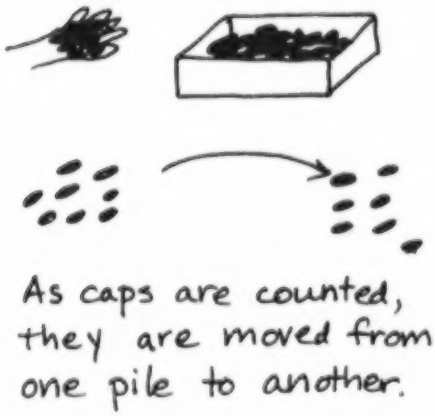
SELECTED ACTIVITIES FOR MATHEMATICS SKILL DEVELOPMENT IN YOUNG CHILDREN

COUNTING

Draw "n" Count I

Use three boxes of numerals. Children draw from the first box as a number to start from. They draw from the second box as a number to count by. A number from the third box is the ending number. The group then counts aloud according to the instructions.





Draw "n" Count II

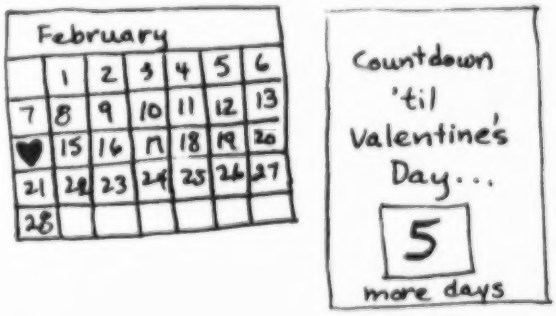
Children take turns drawing a handful of materials from a box or bag, laying the objects on the table, and leading the group to count the objects. Use bottle caps, styrofoam pieces, toothpicks, marbles, and other small objects. The student who leads the counting should move the objects from the original pile to another pile as they are counted. For more than 10 objects, they might make piles of tens and ones to show the place value of the answer.

Counting in Everyday Situations

Take advantage of everyday situations and have children count.

- Use the calendar and count a holiday or vacation.
- Count books in the bookcase.
- Count children wearing a certain color or type clothes.
- See how many crayons the children in the first row have.

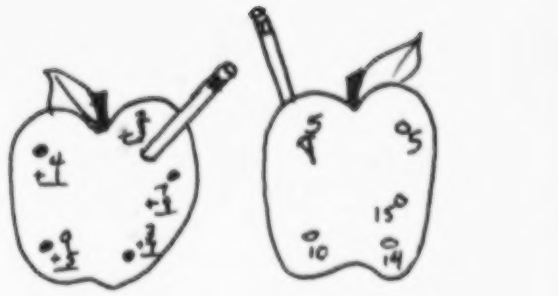
Add to the list of ideas.



BASIC FACTS

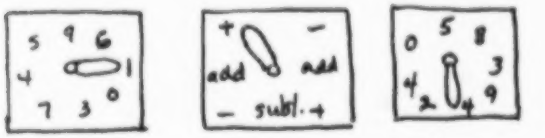
Say It and Check

Children use little patterned devices with problems on front and a hole punched near each. They say the answer and stick their pencil through the hole. On the back (the pencil point marks the spot) children can check their answers. Patterns can be varied to fit seasonal or pupil interests.



front has problems

back has answers -- the pencil marks the spot



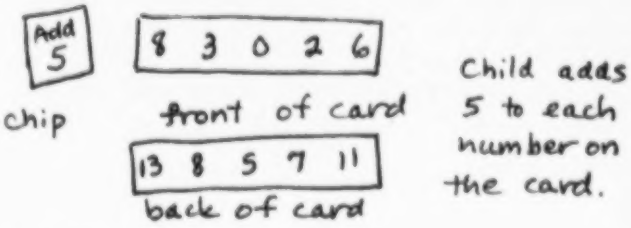
" 1 + 4 " = 5

Spinner Computations

Use the spinners from the concepts lesson plus an operation sign spinner. Let teams of three children take turns spinning, saying the answers quickly and checking themselves.

Chips and Cards

Children work alone or in pairs. They draw a "chip" from a pile and use the number on the chip to perform an operation on each of the numbers on a card. Cards have answers on the back for easy self-checking.

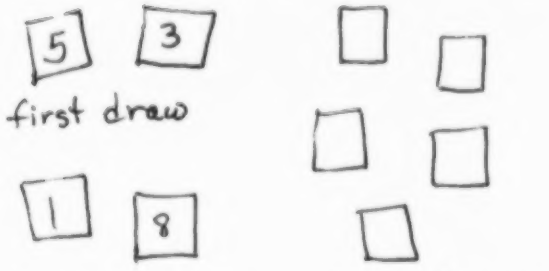


chip

front of card

back of card

Child adds 5 to each number on the card.



first draw

second draw

turned-over cards

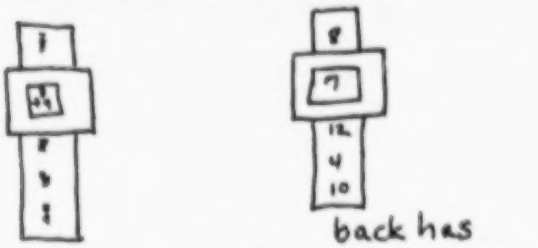
problem :

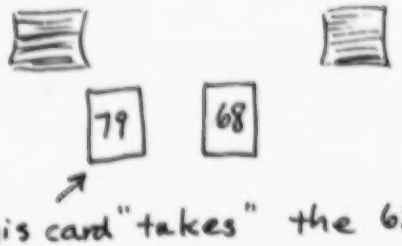
$$\begin{array}{r} 53 \\ -18 \\ \hline 35 \end{array}$$

Subtraction Pairs

Children work in pairs. One draws 2 cards from a pile of cards labeled 0-9. He or she makes the largest possible number from the cards. (If 3 and 5 are drawn, make 53.) The other child draws 2 cards from the pile and makes the smallest possible number. (If 1 and 8 are drawn, make 18, not 81.) Then the pair work together to subtract the smaller from the larger number and check themselves.

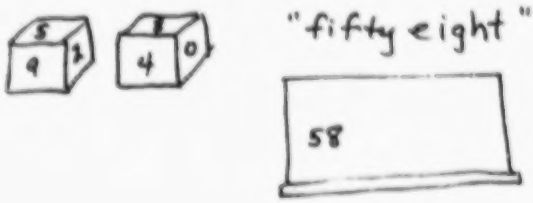
"Battle"
Pairs of children use 2 decks of child- or teacher-made cards. They turn cards over and compare. The child with the larger number "takes" the smaller number. If the numbers are equal, the cards are kept by each. Count cards at the end and determine a winner.





Numeral Viewers
Children work in pairs. A child looks at one side of the card in the viewer and says the number name. The other child checks by reading the answers on the back. Partners should then change roles.

Roll 'n' Say 'n' Write
Work in a small group near the chalkboard. Use number cubes. Roll the cubes and arrange to form a numeral. Let children say the numeral aloud and one child write it on the chalkboard.

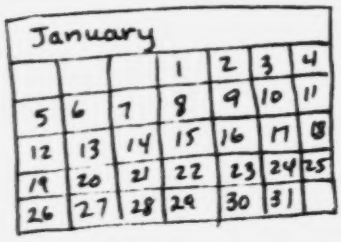


Numbers Relay
Let children sit in rows to have 4-5 teams. A first child from each row comes to the chalkboard and writes a numeral after the teacher or assistant says it slowly. Several children from each row repeat the writing step with different numerals until 4-6 numerals are on the board. The next child from each row comes up and the teacher rereads just one of the numerals and the child points to it. The next child from each row might be asked to read a numeral, or choose the largest or smallest numeral. A last team member might add up all the numerals. Then erase and proceed with more numbers. Keep this moving quickly!

MEASUREMENT SKILLS

Money Clowns

Children work in pairs or individually. They look at the coins pictures on the clown's tongue and say the amount of money. Then they pull the tongue out just a bit to see the answers.



"How many school days this month?"

"Add up the Sundays. Add up the Fridays. Now find the difference."

Find It on the Calendar

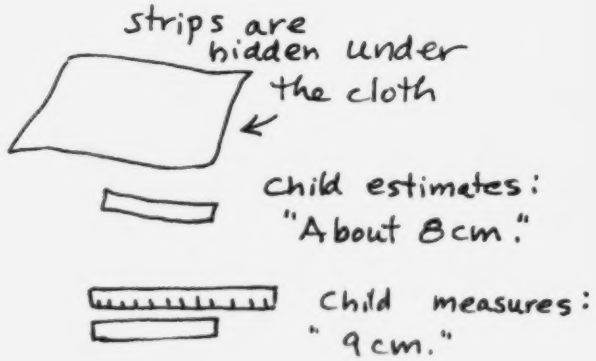
Children work in a large or small or small group with copies of calendars (possibly made by the children). They locate dates on the calendar and solve problems such as these:

- What day of the week is the 18th?
- What date is the second Sunday?
- What date is two weeks from the second Thursday?
- Find all the Saturday dates and add them up.

Guess and Check

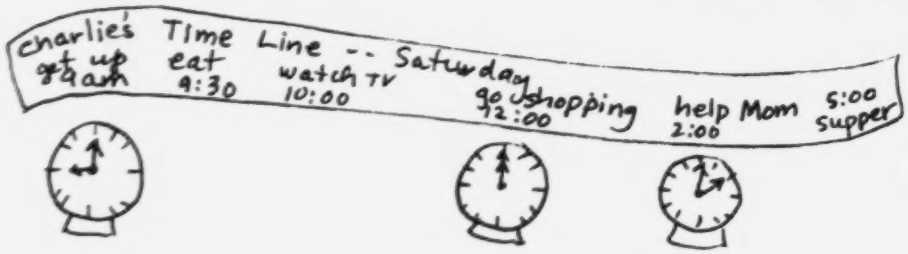
Children work in small groups around a table or on the floor. Place strips of paper (cut in various lengths) under a cloth or larger paper. Children take turns drawing out pieces of paper, extimating the legth, and checking by measuring in centimeters or inches. They could record their results.

strip	estimate	actual
#1	7cm	9cm
#2	8cm	9cm
#3	2cm	3cm



Time Lines

Children work in small groups. Each makes a time line of what he/she did during the previous evening or Saturday. Then they take turns setting model clocks to times shown on their time lines.



PROBLEM SOLVING

25 25 25 1 1 1 1

or

10 10 10 10 5 1 1 1 1 10 10 10

Change and More Change

Work in small groups. Cut pictures and prices of sale items from the newspaper. Let children arrange combinations of coins or bills on the pictures. Encourage various answers. Have children check each other by counting money to see that the total is correct.

Group 1	hand	foot
Andy	12cm	18 cm
DeLisa	11cm	17 cm
MaryAnne	12 cm	15cm
Billy	13 cm	20 cm
Vicki	12cm	18 cm
Whitney	10cm	16 cm

Measure and Make a Chart

Children work in groups of 6-8. They make a chart with measures of classmates' hands and feet. Then the group discusses measurements and tells from the chart information such as:

- Are most peoples' hands or feet longer or are they about the same?
- Whose foot is longest?
- Whose hand is shortest?
- What's the total length of all the feet?

Row Problems

Children work with others from the row (or table) where they sit. They work together to solve problems such as these:

- Do you have more crayons or pencils in your row? How many more?
- Use your first names only. Find the total number of letters used in your first names.
- Measure around your ankles and around your wrists. See which total is more and by how much.

Answers will vary, so let groups share answers and the ways they solved the problems.

Thoughts for leaders are found throughout!

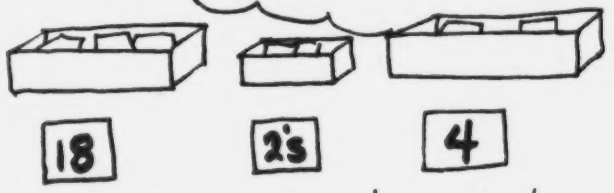
SELECTED ACTIVITIES FOR MATHEMATICS SKILL DEVELOPMENT
IN YOUNG CHILDREN

Try this out with participants -- make middle box of small nos.

COUNTING

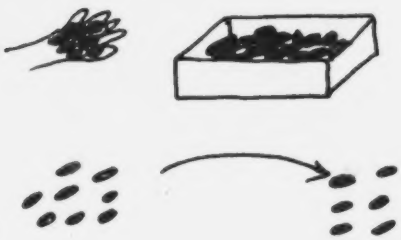
Draw "n" Count I

Use three boxes of numerals. Children draw from the first box as a number to start from. They draw from the second box as a number to count by. A number from the third box is the ending number. The group then counts aloud according to the instructions.



start at 18 -- count by 2's -- to 4

"18, 16, 14, 12, 10, 8, 6, 4"



Stress this step for rational counting

As caps are counted, they are moved from one pile to another.

Draw "n" Count II

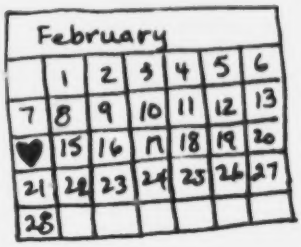
Children take turns drawing a handful of materials from a box or bag, laying the objects on the table, and leading the group to count the objects. Use bottle caps, styrofoam pieces, toothpicks, marbles, and other small objects. The student who leads the counting should move the objects from the original pile to another pile as they are counted. For more than 10 objects, they might make piles of tens and ones to show the place value of the answer.

Counting in Everyday Situations

Take advantage of everyday situations and have children count.

- Use the calendar and count a holiday or vacation.
- Count books in the bookcase.
- Count children wearing a certain color or type clothes.
- See how many crayons the children in the first row have.

Add to the list of ideas.



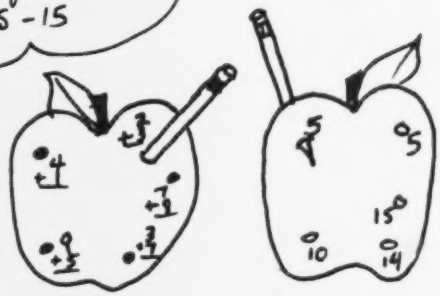
Get participants' ideas here. Add them to the little "slate"... pattern S-12

Several ideas for this on pattern S-15

BASIC FACTS

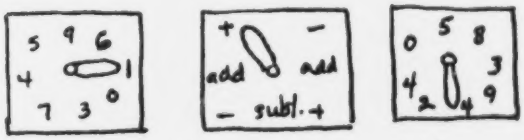
Say It and Check

Children use little patterned devices with problems on front and a hole punched near each. They say the answer and stick their pencil through the hole. On the back (the pencil point marks the spot) children can check their answers. Patterns can be varied to fit seasonal or pupil interests.



front has problems

back has answers -- the pencil marks the spot



" 1 + 4" = 5

Spinner Computations

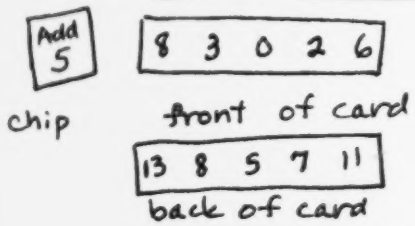
Use the spinners from the concepts lesson plus an operation sign spinner. Let teams of three children take turns spinning, saying the answers quickly and checking themselves.

Pattern S-16 and C-19

make up chips and cards -- no pattern for this simple one is included.

Chips and Cards

Children work alone or in pairs. They draw a "chip" from a pile and use the number on the chip to perform an operation on each of the numbers on a card. Cards have answers on the back for easy self-checking.

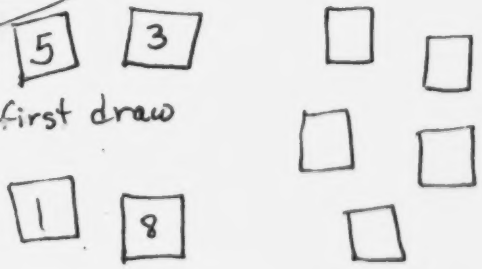


chip

front of card

back of card

Child adds 5 to each number on the card.



first draw

second draw

turned-over cards

problem:

$$\begin{array}{r} 53 \\ -18 \\ \hline 35 \end{array}$$

Subtraction Pairs

Children work in pairs. One draws 2 cards from a pile of cards labeled 0-9. He or she makes the largest possible number from the cards. (If 3 and 5 are drawn, make 53.) The other child draws 2 cards from the pile and makes the smallest possible number. (If 1 and 8 are drawn, make 18, not 81.) Then the pair work together to subtract the smaller from the larger number and check themselves.

Let participants demonstrate this game too.

NUMERALS AND NUMBER RELATIONSHIPS

"Battle"
Pairs of children use 2 decks of child- or teacher-made cards. They turn cards over and compare. The child with the larger number "takes" the smaller number. If the numbers are equal, the cards are kept by each. Count cards at the end and determine a winner.

front has problems back has answers

Roll 'n' Say 'n' Write
Work in a small group near the chalkboard. Use number cubes. Roll the cubes and arrange to form a numeral. Let children say the numeral aloud and one child write it on the chalkboard.

See notes on pattern S-13. Show participants how to write the problems and answers on the strips.

Have volunteers demonstrate this. Keep it moving quickly!

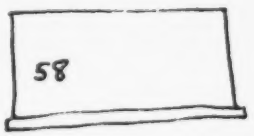
Have 2 participants demonstrate this quick & easy game.

this card "takes" the 68.

Numeral Viewers
Children work in pairs. A child looks at one side of the card in the viewer and says the number name. The other child checks by reading the answers on the back. Partners should then change roles.

Number cube pattern is S-14.

"fifty eight"



Numbers Relay
Let children sit in rows to have 4-5 teams. A first child from each row comes to the chalkboard and writes a numeral after the teacher or assistant says it slowly. Several children from each row repeat the writing step with different numerals until 4-6 numerals are on the board. The next child from each row comes up and the teacher rereads just one of the numerals and the child points to it. The next child from each row might be asked to read a numeral, or choose the largest or smallest numeral. A last team member might add up all the numerals. Then erase and proceed with more numbers. Keep this moving quickly!

MEASUREMENT SKILLS

pattern S-17 -- a sassy character!

Money Clowns

Children work in pairs or individually. They look at the coins pictures on the clown's tongue and say the amount of money. Then they pull the tongue out just a bit to see the answers.

January						
			1	2	3	4
5	6	7	8	9	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30	31	

"How many school days this month?"

"Add up the Sundays. Add up the Fridays. Now find the difference."

Let participants add more problems here.

Find It on the Calendar

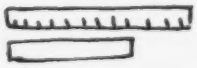
Children work in a large or small or small group with copies of calendars (possibly made by the children). They locate dates on the calendar and solve problems such as these:

- What day of the week is the 18th?
- What date is the second Sunday?
- What date is two weeks from the second Thursday?
- Find all the Saturday dates and add them up.

Guess and Check

Children work in small groups around a table or on the floor. Place strips of paper (cut in various lengths) under a cloth or larger paper. Children take turns drawing out pieces of paper, estimating the length, and checking by measuring in centimeters or inches. They could record their results.

Child estimates: "About 8cm."



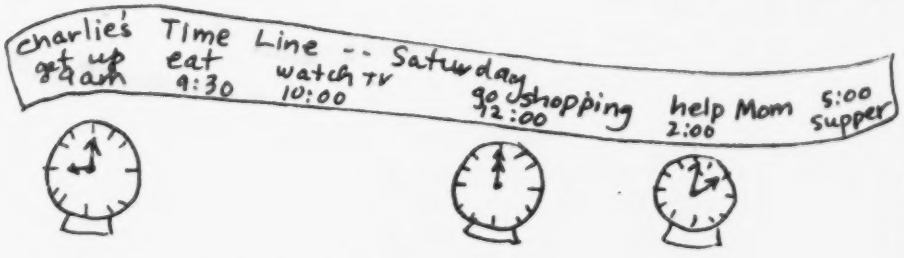
Child measures: "9cm."

Individuals should try this out!

strip	estimate	actual
#1	7cm	9cm
#2	8cm	9cm
#3	2cm	3cm

Time Lines

Children work in small groups. Each makes a time line of what he/she did during the previous evening or Saturday. Then they take turns setting model clocks to times shown on their time lines.

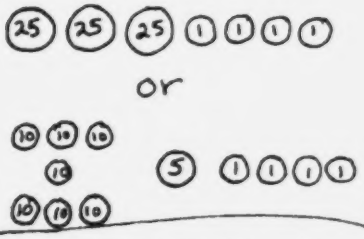


pattern S-18 for clocks.

PROBLEM SOLVING

Change and More Change

Work in small groups. Cut pictures and prices of sale items from the newspaper. Let children arrange combinations of coins or bills on the pictures. Encourage various answers. Have children check each other by counting money to see that the total is correct.



If time permits, let participants clip attractive pertinent ads from the local newspaper ads.

Group 1	hand	foot
Andy	12cm	18 cm
DeLisa	11cm	17 cm
MaryAnne	12 cm	15cm
Billy	13 cm	20 cm
Vicki	12cm	18 cm
Whitney	10cm	16 cm

Measure and Make a Chart

Children work in groups of 6-8. They make a chart with measures of classmates' hands and feet. Then the group discusses measurements and tells from the chart information such as:

- Are most peoples' hands or feet longer or are they about the same?
- Whose foot is longest?
- Whose hand is shortest?
- What's the total length of all the feet?

Participants can try this out -- they'll need a tape measure or ruler

Row Problems

Children work with others from the row (or table) where they sit. They work together to solve problems such as these:

- Do you have more crayons or pencils in your row? How many more?
- Use your first names only. Find the total number of letters used in your first names.
- Measure around your ankles and around your wrists. See which total is more and by how much.

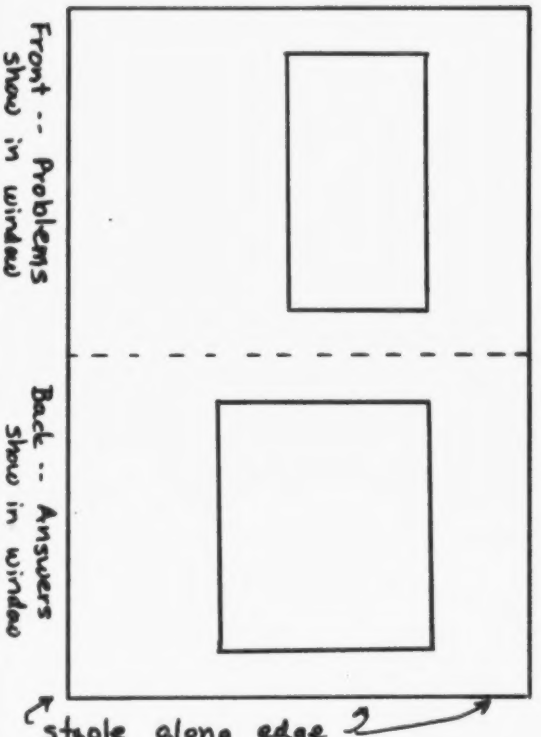
Answers will vary, so let groups share answers and the ways they solved the problems.

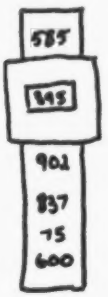
Get other suggestions for problems.

More Ideas for COUNTING in EVERY DAY SITUATIONS

Say the Number	Say the Number	Say the Number
432	880	175
92	179	352
809	17	666
483	295	962
270	592	181
505	611	987

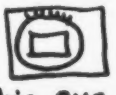
FACT VIEWERS





The child pulls the strip through and tries to answer the question. A second child watches the answers on the back and checks. The strips at the right are just examples. Vary the problems to fit the child's needs.

Decorate the viewer opening if you wish


a big eye

a TV set

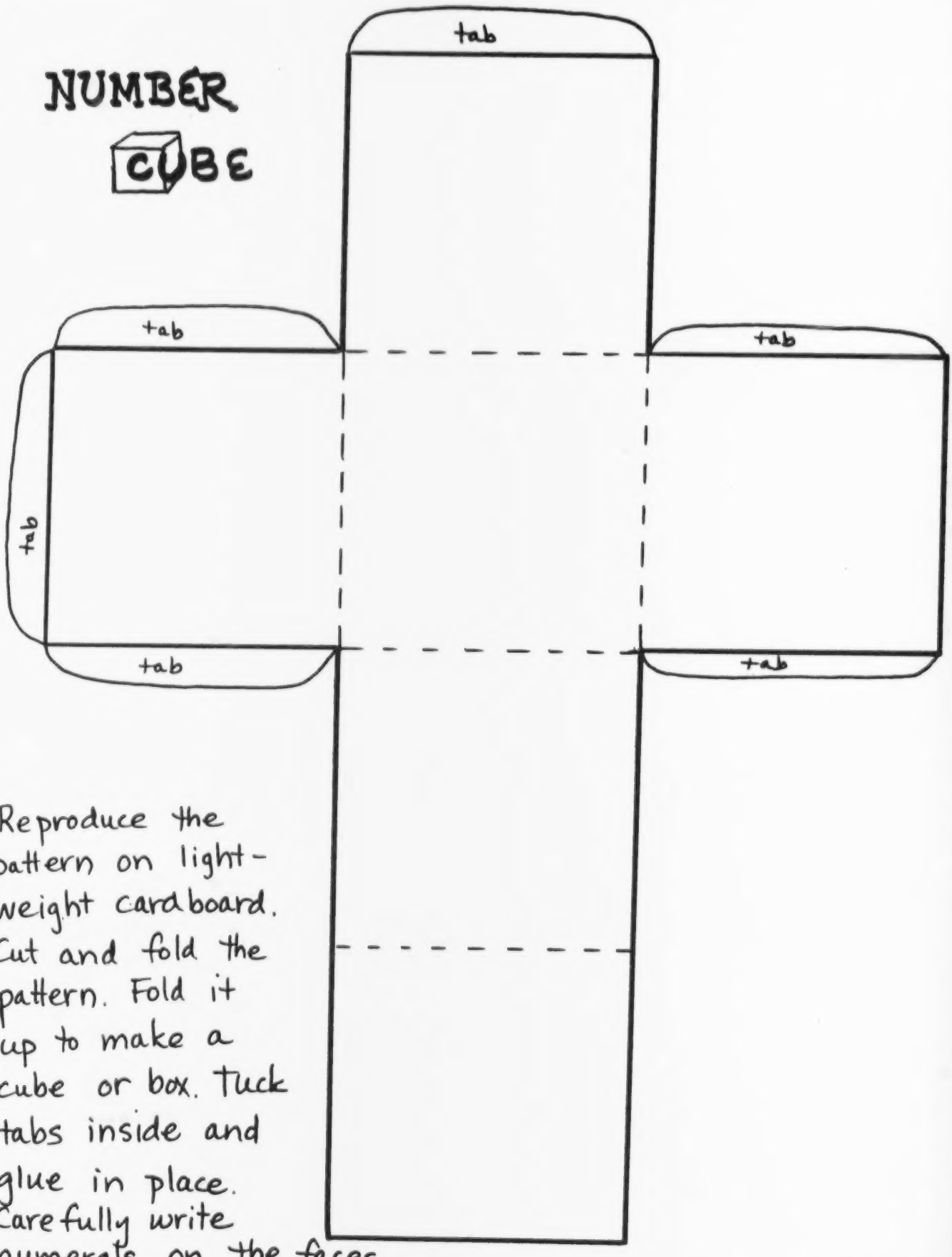
S-13a

- Examples of backs of strips for fact viewers---
  To make more strips, cut the strips and insert into the viewer.
- Write a problem. Turn the viewer over and write the answer.
  Move the strip up and write the next problem and answer. Continue.

Answers to Say the Number	Answers to Say the Number	Answers to Say the Number
1 hundred seventy five	8 hundred eighty	4 hundred thirty two
3 hundred fifty two	1 hundred seventy nine	ninety two
6 hundred sixty six	seventeen	8 hundred nine
9 hundred sixty two	2 hundred ninety five	4 hundred eighty three
1 hundred eighty one	5 hundred ninety two	2 hundred seventy
9 hundred eighty seven	6 hundred eleven	5 hundred five

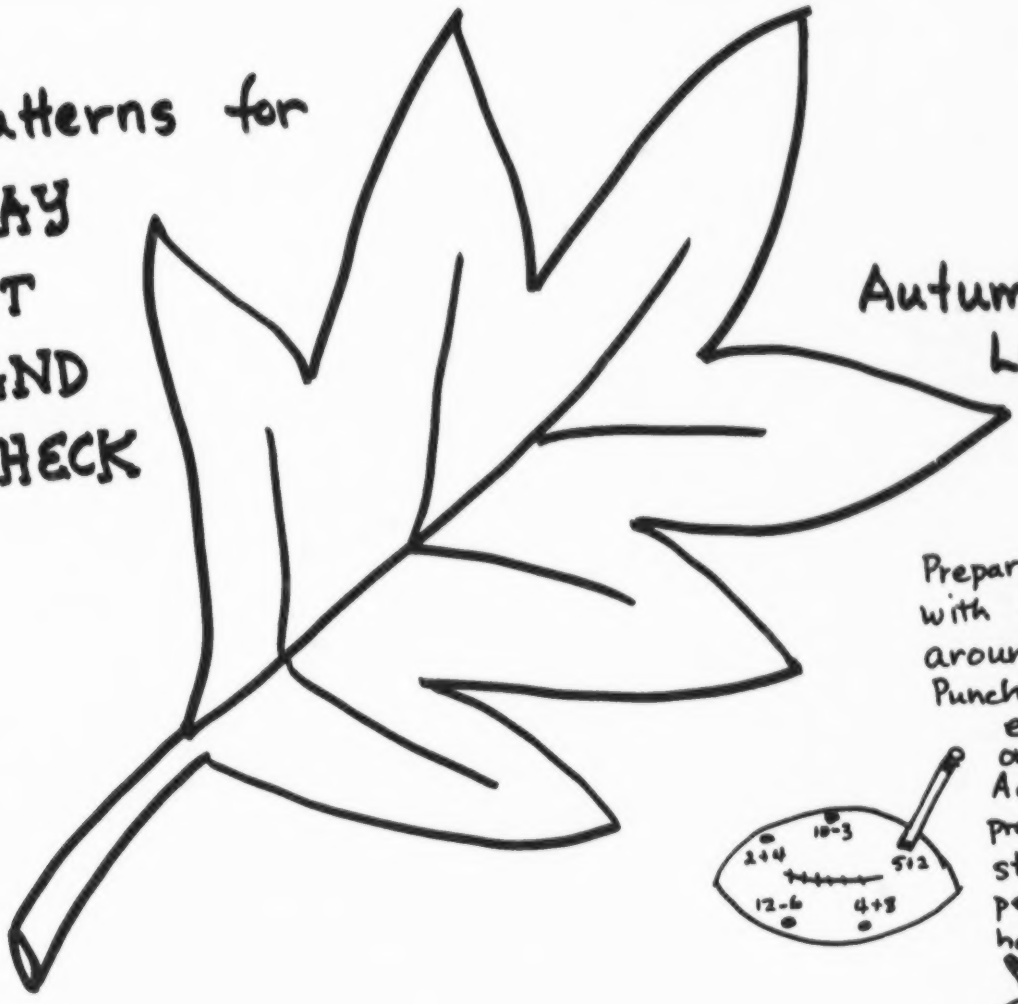
S-13b

NUMBER CUBE



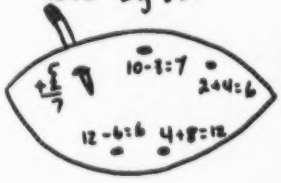
Reproduce the pattern on light-weight cardboard. Cut and fold the pattern. Fold it up to make a cube or box. Tuck tabs inside and glue in place. Carefully write numerals on the faces.

Patterns for SAY IT AND CHECK

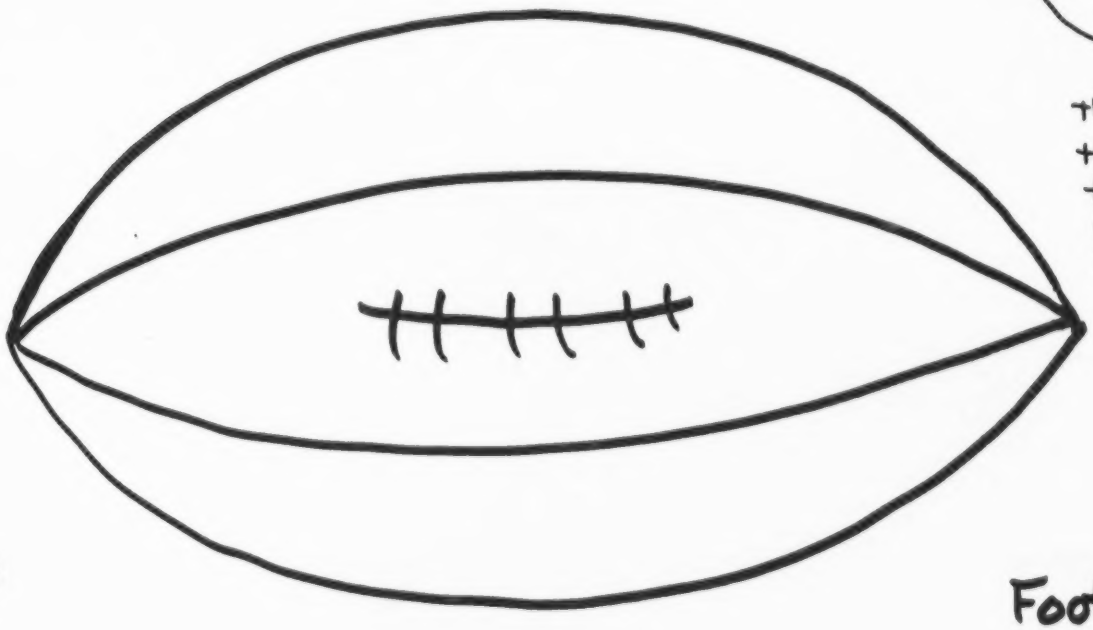


Autumn Leaves

Prepare the patterns with several problems around the edges. Punch holes near each. Write answers on the back. A child tries a problem and sticks a pencil in the hole by it.



the child turns the pattern over to check. the pencil point marks the problem and answer!

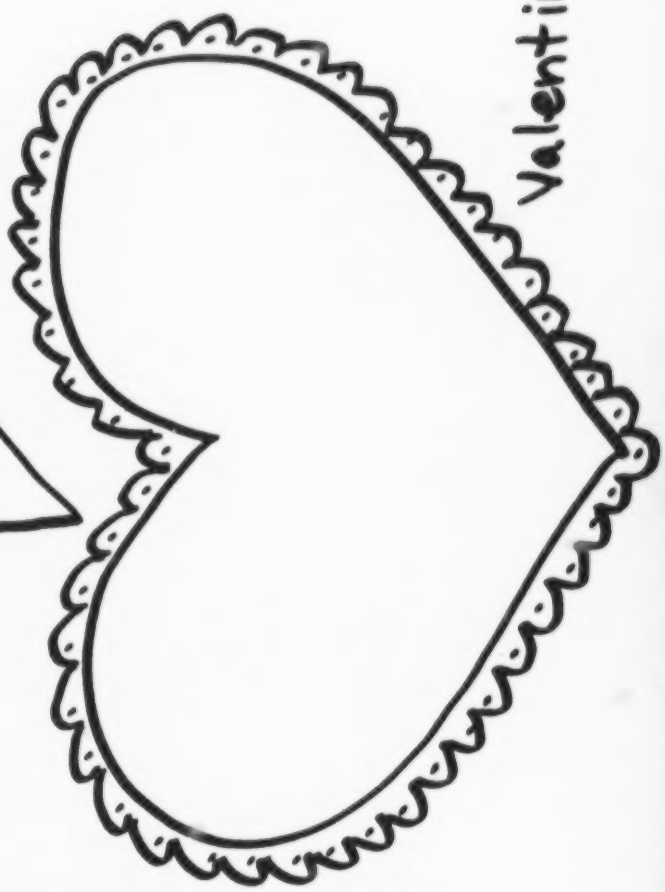


Footballs

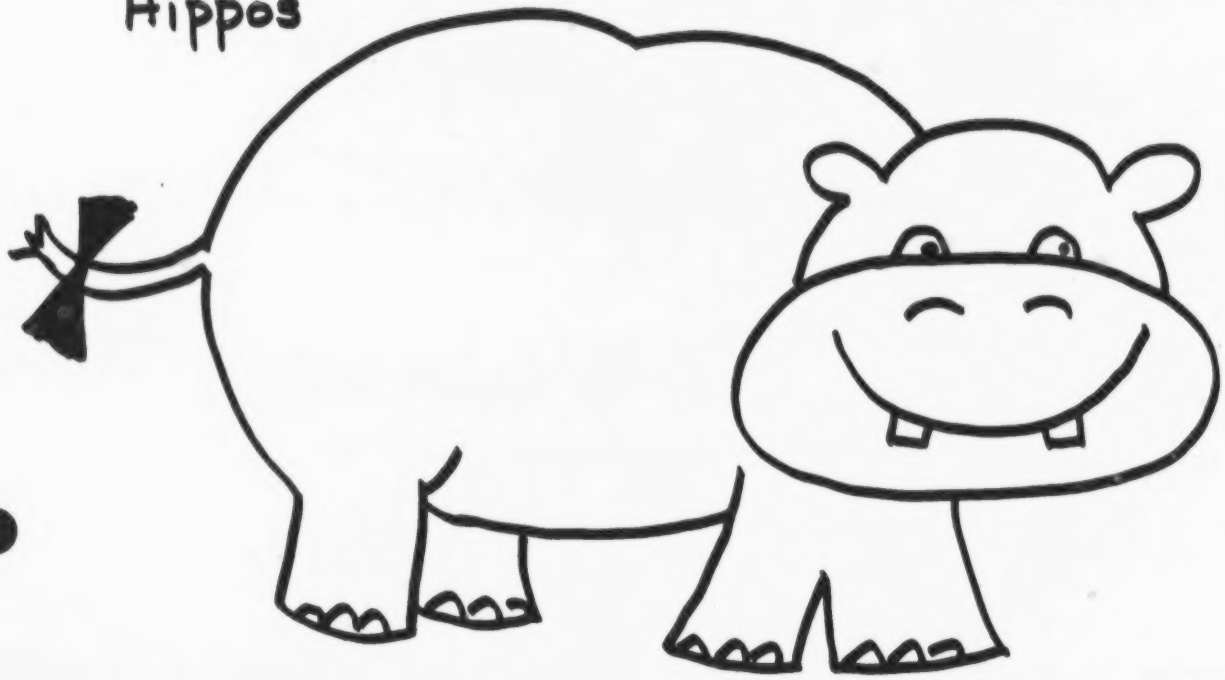
S-15a

Christmas Trees

Valentines

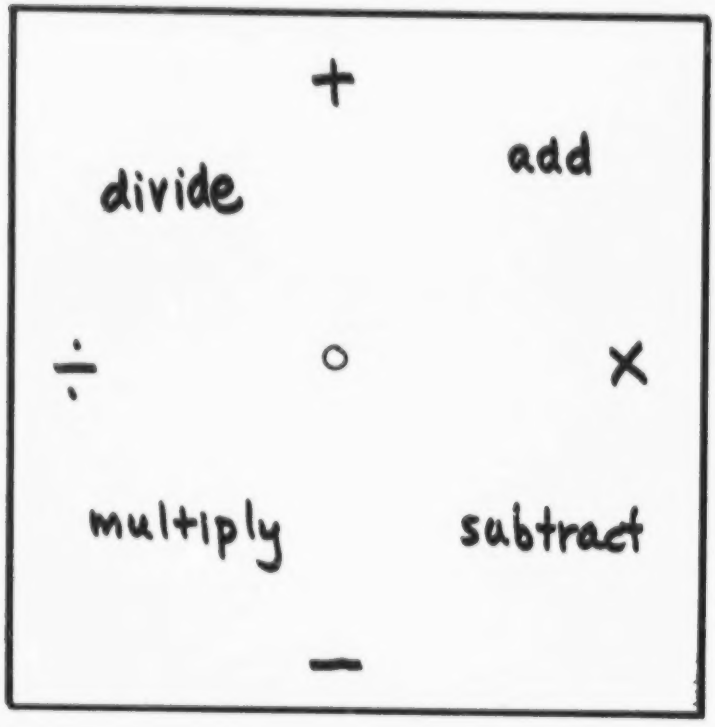


Hippos



OPERATIONS SPINNER

If children need to practice fewer skills, redo the operations signs & words on the dial



Use with the numbers spinners on pattern C - 19.

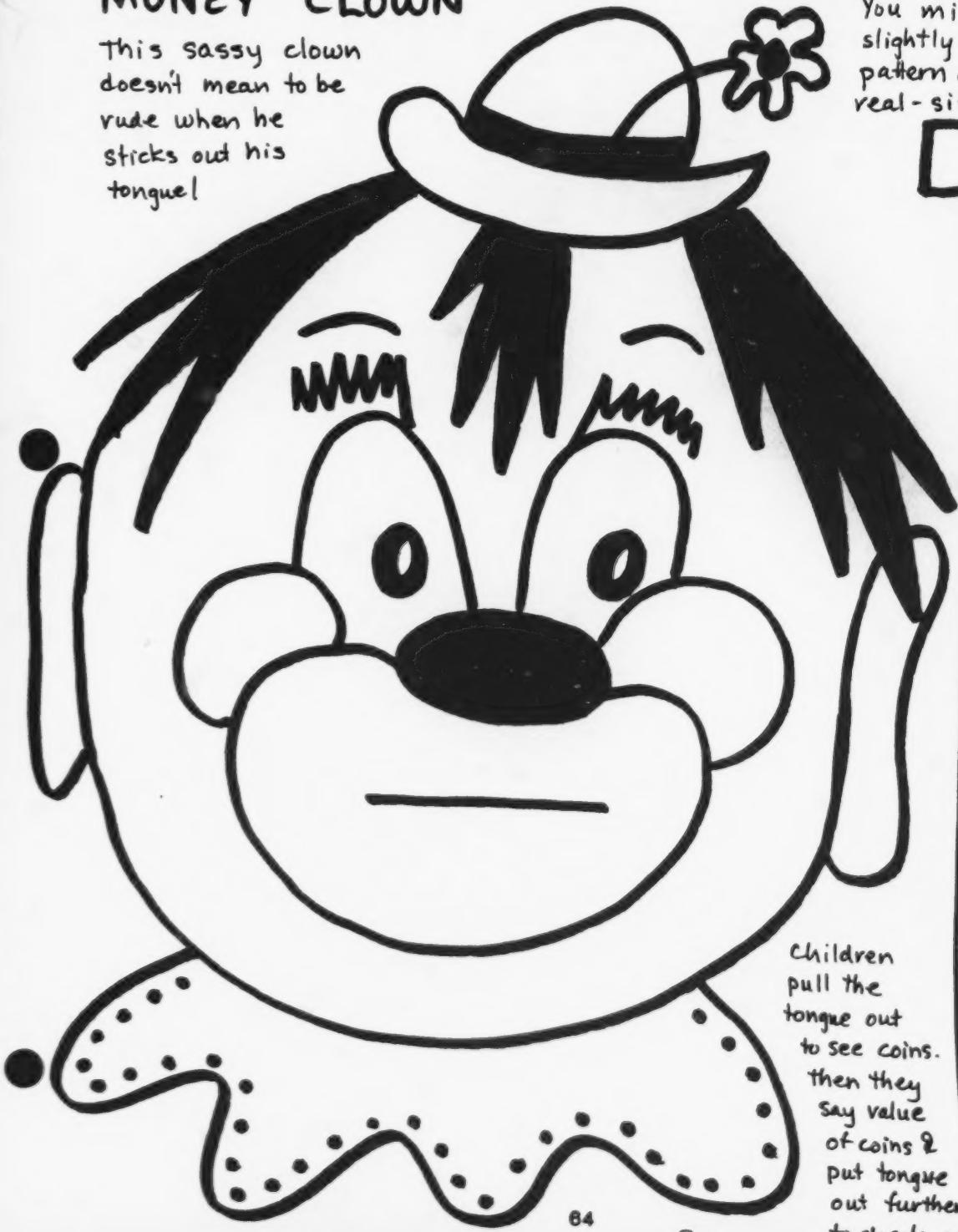
Duplicate and laminate the spinner. Attach a large paper clip to the center with a brad. Make sure the clip spins freely.

5-17

MONEY CLOWN

This sassy clown doesn't mean to be rude when he sticks out his tongue!

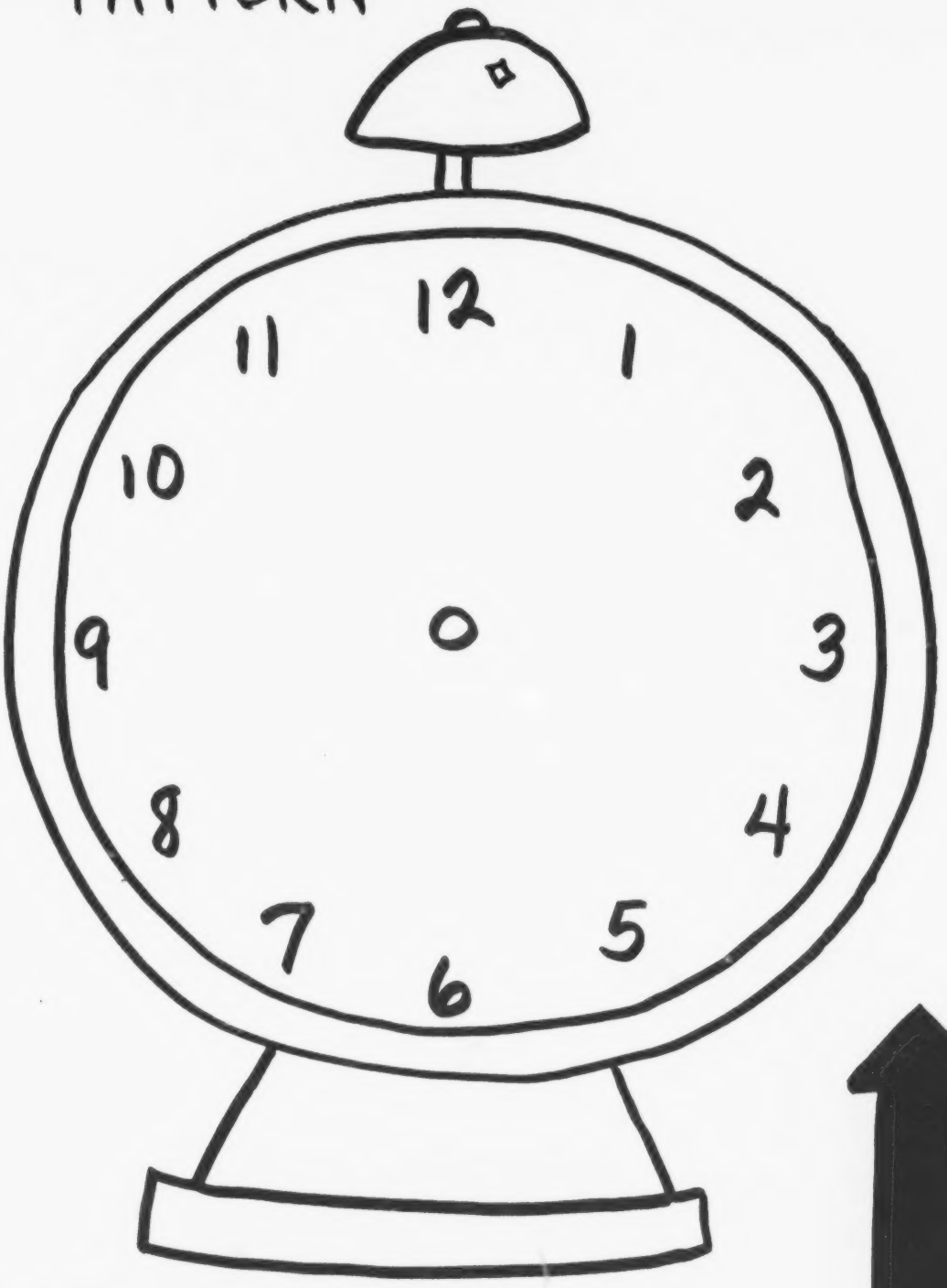
You might also slightly enlarge the pattern and use real-sized coins.



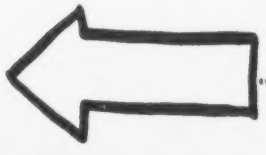
Children pull the tongue out to see coins. then they say value of coins & put tongue out further to check answers.

84

CLOCK PATTERN



"Big" Hand



"Little" Hand

Attach hands with a brad through the center

HAND-IN-HAND: MORE ABOUT CONCEPT AND SKILL DEVELOPMENT IN MATHEMATICS

HAND-IN-HAND: MORE ABOUT CONCEPT AND SKILL DEVELOPMENT IN MATHEMATICS

This workshop session features a review of the interrelated nature of concept and skill development followed by exploration of materials and methods for enhancing development of selected mathematical ideas. Participants will receive information, then extend ideas presented to generate and share more teaching techniques. This session, like the previous ones, could be followed by a time for making classroom materials. The workshop leader should:

1. Review the relationship of mathematics concepts and skills.
2. Lead participants to examine and use teaching techniques to enhance concept and skill development in children.
3. Organize group work in which multigrade teams extend and apply information previously presented and add more ideas for teaching and learning.
4. Prepare materials prior to the presentations:

   Transparencies, S-3, S-4, H-1 through H-4, H-16, H-17.

   Participants' handout H-5.

   Made-up patterns, H-7, H-8, H-11, H-12, H-13.

   Examples of worksheets, H-10, H-15.

   Gather these materials: centimeter rulers or an H-18 pattern for each participant, small objects such as marbles or counters, box or "Mystery Bag," clear or translucent plastic bottle, play dough and a plastic knife, water in a pitcher or rice, measuring cups, coins and several dollar bills, felt board cutouts for subtraction models, triangular flashcards, clothes hanger with clothespins, paper strips in various cm lengths.

As previously described, leaders may want to provide ditto masters or paper for participants to record the ideas they generate for later duplication and distribution. An overhead projector is needed for this presentation.

1. More about concepts and skills. Concepts and skills are interrelated. Previously we used the phrase "hand-in-hand" to point out that in teaching children, neither skills nor concepts can stand alone. (Could use Transparencies S-3 and S-4.) Both must be developed if children are to deal effectively and efficiently with mathematical ideas.

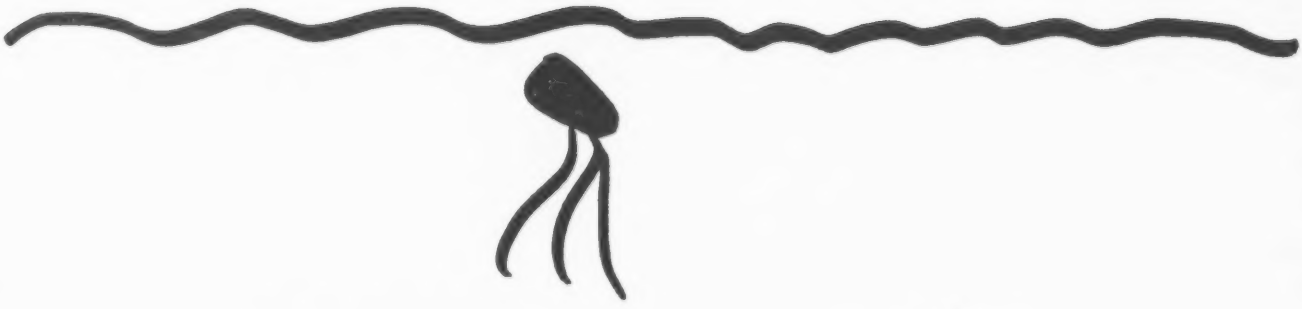
   We can liken concept and skill development to growing plants. Concepts are like good soil and well-developed roots. They provide a firm foundation on which plants grow and thrive. Concepts, like soil and roots, are the base of mathematical ideas (Transparency H-0-2). When plants are healthy, their leaves and flowers grow tall and strong. Leaf and flower growth can be likened to skill development. People can see and appreciate growth and progress of plants above the ground (Transparency overlay H-0-2). But as skills "grow taller" and extend, so should the child's conceptual base. Most plants cannot be healthy without growth both above and below the ground (Transparency overlay H-0-3). Both concepts and skills grow together!

   Concepts and skills are nurtured by constant attention. Like sun helps plants to grow, pupil involvement through expression of ideas, group work, handling of manipulatives, oral and written drill, and project work are necessary for growth of concepts and skills (Transparency overlay H-0-4). Just as plants need water, children need sound teaching methods to ensure their healthy growth in mathematics (Transparency overlay H-0-5).

2. Selected teaching techniques. This workshop will focus on concept and skill development for three mathematics topics — fractions, subtraction, and length. All of these topics are introduced early in the curriculum and are extended through the upper grades. Good "grounding" in these areas helps children to understand and interpret their current worlds and also to prepare for future academic work.

Use participant Handout H-5 and the leader's copy with notes (H-6). Present the ideas and materials involving the participants in the activities as you go along. Invite the participants to jot down ideas they may have for extending and expanding the activites.

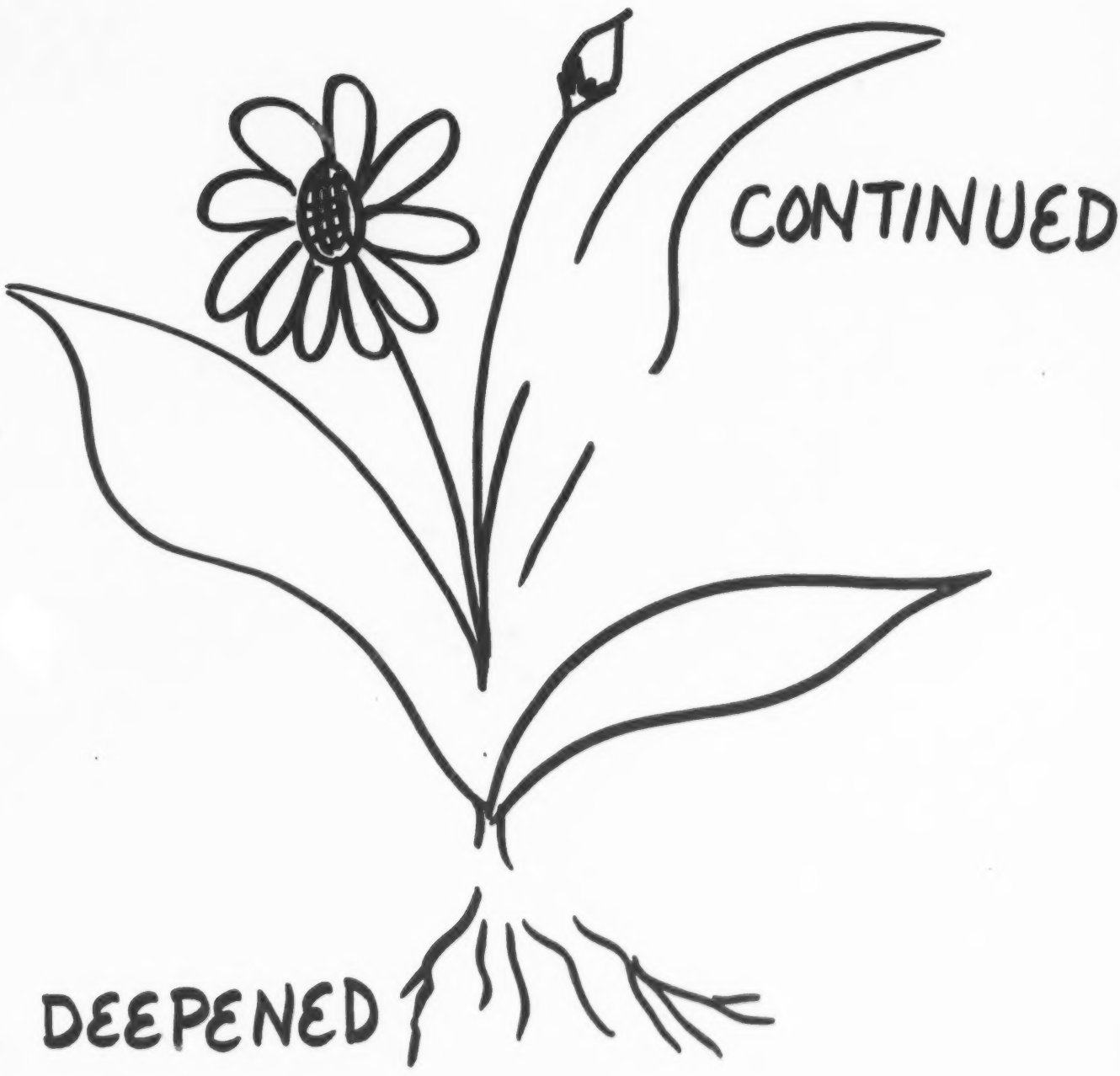
3. Group work. Organize multilevel groups. Have some kindergarten, first, second, and third grade teachers and teaching assistants work together. Groups of five to eight persons may work well. Have each group select a math area and think of more ideas for extending children's concepts and skills. Some groups may enlarge the three areas already started — fractions, subtraction, or length — while others may wish to take other areas such as number relations, problem solving, or working with time. Have the groups plan to share their ideas orally after a short period of time. You may wish to let each group make notes and sketches for future duplication and distribution. Be sure each group plans concept and skill development activities for each grade level represented (that is, some beginning activities for kindergarten, and ways to deepen and expand the ideas and skill in the upper grades).



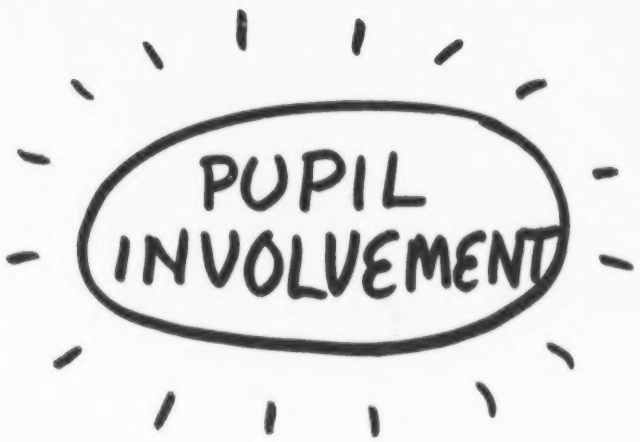
CONCEPTUAL BASE

H1

GROWTH OF SKILLS



H-O-3


PUPIL
INVOLVEMENT

SOUND
TEACHING

SOME TEACHING-LEARNING STRATEGIES
FOR SELECTED AREAS OF MATH FOR YOUNG CHILDREN

FRACTIONS

Concepts	Skills
Fractions can tell us about parts of a whole.	
Show children how to cut up various objects to show halves and fourths. "When we have 2 equal parts, we call them halves. When we have 4 equal parts, we call each one a fourth." Use playdough balls and "hot dogs," paper rectangles and circles, and cookies or crackers to cut into parts.	Have children cut up or break up objects into fourths and halves. Let them place the fractional parts side by side or atop one another to show that their fractional parts are equal. Ask children "Show me half" or "Tell me the name of this part."
Have children cut out and color the pizzas (Pattern H-7). Talk about the fractional names for 1, 2, and 3 parts out of 4 (1/4, 2/4 or 1/2, 3/4). Ask questions such as "If you were hungry, would you want 1/4 or 4/4?" Let the children show their answers. Talk about 4/4 equaling the whole.	Play "Show Me." Call out fractional parts and let the children show them with the pizza pieces. Write fractions on the board and have children show these with pizza pieces. Have children label each piece "1/4" on the back.

Repeat similar activities with thirds and fifths (Patterns H-8—cakes and candy canes.)

Fractions can also designate parts of groups.	
Show the fish models (Pattern H-9). Make little stories such as "One little fish out of three is red. What fraction is this? One more fish came swimming by. It was blue. Now what fraction of the fish were blue?" Present "one out of four" as another way to say 1/4.	Have children tell and write fractions for various arrangements of fish and ask classmates for the appropriate fractions. Use Worksheet (H-10).
Invite groups of 4 or 5 children to come forward and make problems such as "What fraction of the children are wearing red (wearing barettes, sandals, etc.)?" Present a few situations where all of the children (4/4 or 5/5) demonstrate the characteristic.	Present fraction cards and have children take turns picking the appropriate situations where 2/4 = 1/2 or 2/6 = 1/3, guide children to see that both fractional names fit.
Have children work in small groups. Have them fill in squares on a bar graph format (Pattern H-11) on topics such as "Would you rather drink orange juice or apple juice?" and "Which snack would you choose?"	Children tell about graph results. They tell topics and fractions of participants who chose each category. If none chooses a category, introduce its fraction, such as 0/5 or 0/8. Label graphs with fractions.

Fractions can show division, too.

Work with a small group and a sack of materials such as marbles or plastic counters. Specify a number of "shares" or groups in which to divide materials. Pass the materials out into styrofoam trays (like dealing cards) so that each "share" is equal. If there are four shares, each is 1/4 of the total and so on. Call any leftovers the remainder.

Say and write fractions for each share.

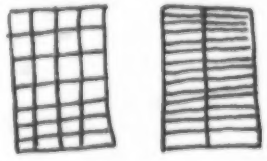
Use the "Dogs and Bones" or "Bears and Hamburgers" patterns (Patterns H-12 and H-13). Choose enough animals to show thirds, fourths, or fifths. Choose a "handful" of bones or hamburgers and let the children distribute them evenly.

Say and write the fractions for each situation. Ask children to show situations such as "Give this bear half of these (12) hamburgers and the other bear the other half," and "Arrange 1/5 of the bones by each dog."

Use a real sheet cake or pan of finger jello (Recipe H-14). Talk with the children and sketch fractions of the food starting with halves and thirds and fourths. Count the number of people who will actually share the food. Plan for equal shares — each will get 1/28 for example. Plan, cut, and share the food.

Let children make sketches of simple fractions and also some ways to cut a rectangle into larger numbers of pieces (25, 28, 30, etc.). Name the fractions. Label some sketches with the appropriate fractions.

some ways to cut 28ths



People use fractional measures.

Fill a plastic bottle or box with colored water, rice, or sand to show fractions such as halves, thirds, and fourths. Use a permanent marker to label fractions on the container.

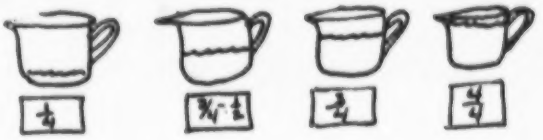
"one half"

2 liter soft drink bottle with top cut off.

Have children fill the container to given levels. Have children match fraction cards to various levels in the container. Have children make and label sketches of their results on a worksheet (but don't use the worksheet without actually doing the experience). (Worksheet H-15)

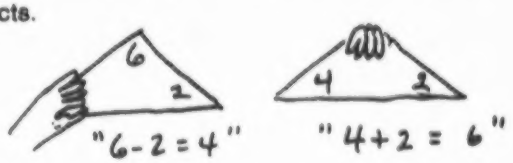
Show 4 identical measuring cups side by side. Let children fill them to various levels, then arrange them in order from smallest to largest volumes. Write some number sentences such as $1/3 < 3/4$ and $3/4 > 1/4$ (but don't expect all children to master this in a formal way).

Let children make labels for cups and place them beside the cups. Have children say the names of the fractions shown.



SUBTRACTION

Concepts	Skills
Subtraction has many levels.	
Illustrate each model (transparency H-16) several ways. For example, in the take-away model use various counters and ask children to take some away. For the more needed model, have children show situations with coins. Have children also show comparison situations with counters. For partitioning situations use actual children to illustrate. Ask the children to make partitioning problems, too.	Use oral written take-away problems. Children follow the directions and tell and write the answers. Use oral and written more needed problems. Children tell and write the answers. Use written and oral comparison problems. Use oral questions based on characteristics of children. Have children write and tell answers.
Addition and subtraction facts are related.	
For related facts of 6, use 6 felt circles — 2 blue and 4 red. Arrange them various ways. Pose problems and help children write number sentences. "Let's line them up. How many in all?" "Six circles, two blue, the rest red. How many red?" and so on. . .	Children make and use triangular flashcards to practice related facts.
Use an addition chart (transparency H-17). Help children fill in sums. Discuss patterns found in the chart.	Show children how to find subtraction answers in the chart. Have them demonstrate how to find answers.
Use clothespins on a hanger and show children 4 related facts such as $3+6=9$, $6+3=9$, $9-6=3$ and $9-3=6$. Let children show other groups of 4 related facts.	Have children record work as counters are munipulated. Give children one fact and let them write 3 related facts.



Given $2+8=10$,

children write
$8+2=10$
$10-2=8$
$10-8=2$

In some subtraction problems, regrouping is necessary.

Use counters to illustrate place value in problems without regrouping (78-53 or 25-12). Now use counters to illustrate that in 78-59, there are not enough ones to take away 9, so one group of ten is regrouped to make 10 ones (plus the 8 ones already there), then we can proceed with the subtraction. Have children help to demonstrate the regrouping in problems such as 92-18, 80-66, and 72-69.

Have children tell how to regroup in 72=60+12, 68=50+18, or 91=80 +11. Have children show on the chalkboard or on paper how to regroup in:

72	68	91
-29	-29	-29

Complete each subtraction problem.

Use real dollar bills, dimes, and pennies to illustrate regrouping. Review that $1.00 = 10 dimes and 1 dime = 10 pennies. Start with various amounts and have children use money to show regroupings. For example: $1.18 can be regrouped as 1 dollar, no dimes, and 18 pennies. $2.53 can be regrouped as 1 dollar, 14 dimes, and 13 pennies.

Have children start with a number of dollars and take away a lesser amount. Show and record work. For example:

$1.00	$5.00	$4.00
-.53	-1.89	-2.50

Use coins and count out the change.

MEASUREMENT: LENGTH

Concepts	Skills

Longer and shorter, taller and shorter.

Work with volunteer children to compare lengths, asking questions such as these: Who's taller, shorter, in between? Whose belt is longer? Whose hair is the shortest? Explore other comparison ideas: Whose stripes (on shirt) are widest? Which book is thickest? Thinnest?

Children make oral and written sentences to compare characteristics on classmates.

Have children arrange paper strips in order from tallest to shortest. Direct children to pile blocks or crayon boxes in tasks such as these: Make a stack just as tall as this one. Make your pile shorter than this one.

Children cut, arrange, and describe materials.

Play "Longer Than." Start with a fairly short object and let a first child find something shorter or taller than the object and arrange it next to the first object. Then a second child finds another object longer than an already collected object and arranges it in order. Continue.

Children use descriptive words such as longer, shorter, about the same length to describe the relationships of objects.

Work with a single unit of measure.

(Although centimeters are presented here, adults could use the same procedures to work with inches.)

Use a simple ruler marked only in cm (not mm) (Pattern H-18). Have the children examine and talk about the length of 1 cm. Have them show with their fingertips about how long a cm is. Have children demonstrate lengths such as 5 cm, 10 cm, 8 cm, 2 cm and check those lengths with the ruler.

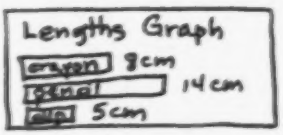
Have children estimate the lengths of small objects such as books, crayons, pencils, notecards, or paper clips. Then they use rulers to measure the lengths of the objects, recording their work.

Children draw paper strips out of a "Mystery Bag" or box. They feel each strip, then estimate its length.

Children record estimates, then use a ruler to measure each strip and record their work.

Children work in groups of 3 or 4. Each group gathers several small objects, then cuts paper strips to the lengths of each. They paste the strips onto paper to create a bar graph. Each group should present and interpret its graph.

Children measure each strip. They label the strips and entitle their graph.



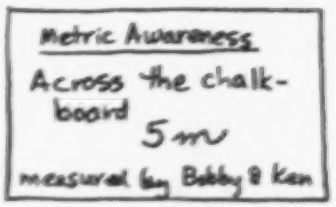
Different measures are more appropriate than others for different lengths.

After exposure to centimeters, meters, and kilometers, pose questions about the most appropriate measure for each. Examples: length of your desk top, your height, measurement around you neck, distance to _____ (some appropriate distance in town), height of a tree, distance from classroom to principal's office.

Children choose the appropriate measure and explain why they chose it.

After the preceeding discussion, assign teams of children to choose an appropriate measuring tool (cm ruler or meter stick) and actually measure some distances. Discuss how one might measure a distance in km. If possible, take the children for a 1 km walk.

Groups measure assigned distances. If possible, they label each with a metric awareness sign.



more thoughts for leaders

SOME TEACHING-LEARNING STRATEGIES
FOR SELECTED AREAS OF MATH FOR YOUNG CHILDREN

FRACTIONS

Concepts	Skills
Fractions can tell us about parts of a whole.	
Show children how to cut up various objects to show halves and fourths. "When we have 2 equal parts, we call them halves. When we have 4 equal parts, we call each one a fourth." Use playdough balls and "hot dogs," paper rectangles and circles, and cookies or crackers to cut into parts.	Have children cut up or break up objects into fourths and halves. Let them place the fractional parts side by side or atop one another to show that their fractional parts are equal. Ask children "Show me half" or "Tell me the name of this part."
Have children cut out and color the pizzas (Pattern H-7). Talk about the fractional names for 1, 2, and 3 parts out of 4 (1/4, 2/4 or 1/2, 3/4). Ask questions such as "If you were hungry, would you want 1/4 or 4/4?" Let the children show their answers. Talk about 4/4 equaling the whole.	Play "Show Me." Call out fractional parts and let the children show them with the pizza pieces. Write fractions on the board and have children show these with pizza pieces. Have children label each piece "1/4" on the back.

use cutouts & real objects to demonstrate

Try this part out

Repeat similar activities with thirds and fifths (Patterns H-8--cakes and candy canes).

Fractions can also designate parts of groups.

Show the fish models (Pattern H-9). Make little stories such as "One little fish out of three is red. What fraction is this? One more fish came swimming by. It was blue. Now what fraction of the fish were blue?" Present "one out of four" as another way to say 1/4.	Have children tell and write fractions for fish stories. Let children show various arrangements of fish and ask classmates for the appropriate fractions. Use worksheet (H-10).
Invite groups of 4 or 5 children to come forward and make problems such as "What fraction of the children are wearing red (wearing barettes, sandels, etc.)?" Present a few situations where all of the children (4/4 or 5/5) demonstrate the characteristic.	Present fraction cards and have children take turns picking the appropriate card for each situation. For situations where 2/4 = 1/2 or 2/6 = 1/3, guide children to see that both fractional names fit.

use the fish cutouts on a flannel board

Let 2 participants role-play this

Try this out too -- let participants suggest questions based on the characteristics of those standing

Make up the graph and let a few participants mark the graph. Let others ask questions.

Have children work in small groups. Have them fill in squares on a bar graph format (pattern H-11) on topics such as "Would you rather drink orange juice or apple juice?" and "Which snack would you choose?"

Children tell about graph results. They tell topics and fractions of participants who chose each category. If none chooses a category, introduce its fraction, such as 0/5 or 0/8. Label graphs with fractions.

Fractions can show division too.

Work with a small group and a sack of materials such as marbles or plastic counters. Specify a number of "shares" or groups in which to divide materials. Pass the materials out into styrofoam trays (like dealing cards) so that each "share" is equal. If there are four shares, each is 1/4 of the total and so on. Call any leftovers the remainder.

Say and write the fractions for each share.

Let some participants demonstrate this.

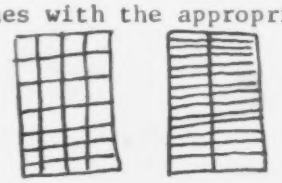
Use the "Dogs and Bones" or "Bears and Hamburgers" patterns (Patterns H-12 and H-13). Choose enough animals to show thirds, fourths, or fifths. Choose a "handful" of bones or hamburgers and let the children distribute them evenly.

Say and write the fractions for each situation. Ask children to show situations such as "Give this bear half of these (12) hamburgers and the other bear the other half," and "arrange 1/5 of the bones by each dog."

Use a real sheet cake or pan of finger jello (Recipe H-14). Talk with the children and sketch fractions of the food starting with halves and thirds and fourths. Count the number of people who will actually share the food. Plan for equal shares--each will get 1/28 for example. Plan, cut, and share the food.

Let children make sketches of simple fractions and also some ways to cut a rectangle into larger numbers of pieces (25, 28, 30, etc.). Name the fractions. Label some sketches with the appropriate fractions.

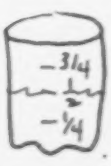
some ways to cut 28ths



People use fractional measures.

Fill a plastic bottle or box with colored water, rice, or sand to show fractions such as halves, thirds, and fourths. Use a permanent marker to label fractions on the container.

Have children fill the container to given levels. Have children match fraction cards to various levels in the container. Have children make and label sketches of their results on a worksheet (but don't use the worksheet without actually doing the experience). (Worksheet H-15)



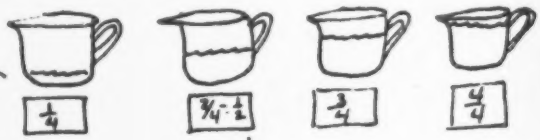
"one half"

2 liter soft drink bottle with top cut off.

Let participants share some way to keep the work area neat when using water, rice or sand.

Show 4 identical measuring cups side by side. Let children fill them to various levels, then arrange them in order from smallest to largest volumes. Write some number sentences such as $1/3 < 3/4$ and $3/4 > 1/4$ (but don't expect all children to master this in a formal way).

Let children make labels for cups and place them beside the cups. Have children say the names of the fractions shown.



Water colored with a little food color makes this especially vivid!

SUBTRACTION

Concepts

Skills

Subtraction has many models.

Take time & talk about each model

Illustrate each model (transparency H-16) several ways. For example, in the take-away model use various counters and ask children to take some away. For the more needed model, have children show situations with coins. Have children also show comparison situa- with counters. For partitioning situations use actual children to illustrate. Ask the children to make partitioning problems too.

Use oral and written take-away problems. Children follow the directions and tell and write the answers.

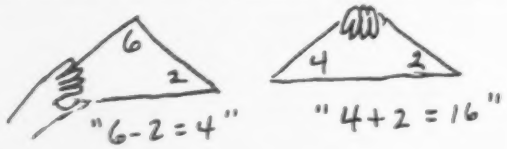
Use oral and written more needed problems. Children tell and write the answers.

Use written and oral comparison problems. Use oral questions based on characteristics of children. Have children write and tell answers.

Addition and subtraction facts are related.

For related facts of 6, us 6 felt circles--2 blue and 4 red. Arrange them various ways. Pose problems and help children write number sentences. "Let's line them up. How many in all?" "Six circles, two blue, the rest red. How many red?" and so on.

Children make and use triangular flashcards to practice related facts.



Use an addition chart (transparency H-17). Help children fill in sums. Discuss patterns found in the chart.

Show children how to find subtraction answers in the chart. Have them demonstrate how to find answers.

Highlight some patterns on the overhead projector as participants describe them.

Show how to hold the flash cards.

Use clothespins on a hanger and show children 4 related facts such as 3+6 = 9, 6+3 = 9, 9-6 = 3 and 9-3 = 6. Let children show other groups of 4 related facts.

3 + 6 = 9

9 together, move 6," take them away", leaves 3

Have a volunteer move clothespins to show various facts.

Have children record work as counters are manipulated. Give children one fact and let them write 3 related facts.

Given $\boxed{2+8=10}$,

children write

8+2=10
10-2=8
10-8=2

Another volunteer can record related facts.

In some subtraction problems, regrouping is necessary.

Use counters to illustrate place value in problems without regrouping (78-53 or 25-12). Now use counters to illustrate that in 78-59, there are not enough ones to take away 9, so one group of ten is regrouped to make 10 ones (plus the 8 ones already there), then we can proceed with the subtraction. Have children help to demonstrate the regrouping in problems such as 92-18, 80-66, and 72-69.

Actually show these problems moving & regrouping counters

Have children tell how to regroup in 72 = 60+12, 68 = 50+18, or 91 = 80+11. Have children show on the chalkboard or on paper how to regroup in:

72	68	91
-29	-29	-29

Complete each subtraction problem.

Use real dollar bills, dimes, and pennies to illustrate regrouping. Review that $1.00 = 10 dimes and 1 dime = 10 pennies. Start with various amounts and have children use money to show regroupings. For example: $1.18 can be regrouped as 1 dollar, no dimes, and 18 pennies. $2.53 can be regrouped as 1 dollar, 14 dimes, and 13 pennies.

Have children start with a number of dollars and take away a lesser amount. Show and record work. For example:

$1.00	$5.00	$4.00
-.53	-1.89	-2.50

Use coins and count of the change.

Try these out too using real money!

MEASUREMENT: LENGTH

Concepts

Skills

Longer and shorter, taller and shorter.

Work with volunteer children to compare lengths, asking questions such as these: Who's taller, shorter, in between? Whose belt is longer? Whose hair is the shortest? Explore other comparison ideas: Whose stripes (on shirt) are widest? Which book is thickest? Thinnest?

Children make oral and written sentences to compare characteristics on classmates.

Have some participants stand up & let others pose various questions.

Have children arrange paper strips in order from tallest to shortest. Direct children to pile blocks or crayon boxes in tasks such as these: Make a stack just as tall as this one. Make your pile shorter than this one.

Children cut, arrange, and describe materials.

Show how to do this with real materials.

Play "Longer Than." Start with a fairly short object and let a first child find something shorter or taller than the object and arrange it next to the first object. Then a second child finds another object longer than an already collected object and arranges it in order. Continue.

Children use descriptive words such as longer, shorter, about the same length to describe the relationships of objects.

Play this game for a few "turns"

Work with a single unit of measure.
(Although centimeters are presented here, adults could use the same procedures to work with inches.)

Use a simple ruler marked only in cm (not mm) (Pattern H-18). Have the children examine and talk about the length of 1 cm. Have them show with their fingertips about how long a cm is. Have children demonstrate lengths such as 5 cm, 10 cm, 8 cm, 2 cm and check those lengths with the ruler.

Have children estimate the lengths of small objects such as books, crayons, pencils, notecards, or paper clips. Then they use rulers to measure the lengths of the objects, recording their work.

"2cm"

Practice this one too!

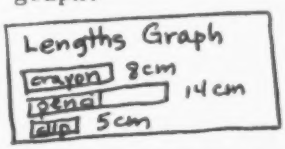
Children draw paper strips out of a "Mystery Bag" or box. They feel each strip, then estimate its length.

Children record estimates, then use a ruler to measure each strip and record lengths.

Try this out or present such a graph made by children

Children work in groups of 3 or 4. Each group gathers several small objects, then cuts paper strips to the lengths of each. They paste the strips onto paper to create a bar graph. Each group should present and interpret its graph.

Children measure each strip. They label the strips and entitle their graph.



Different measures are more appropriate than others for different lengths.

After exposure to centimeters, meters, and kilometers, pose questions about the most appropriate measure for each. Examples: length of your desk top, your height, measurement around your neck, distance to _____ (some appropriate distance in town), height of a tree, distance from classroom to principal's office.

Children choose the appropriate measure and explain why they chose it.

Encourage participants to give other examples.

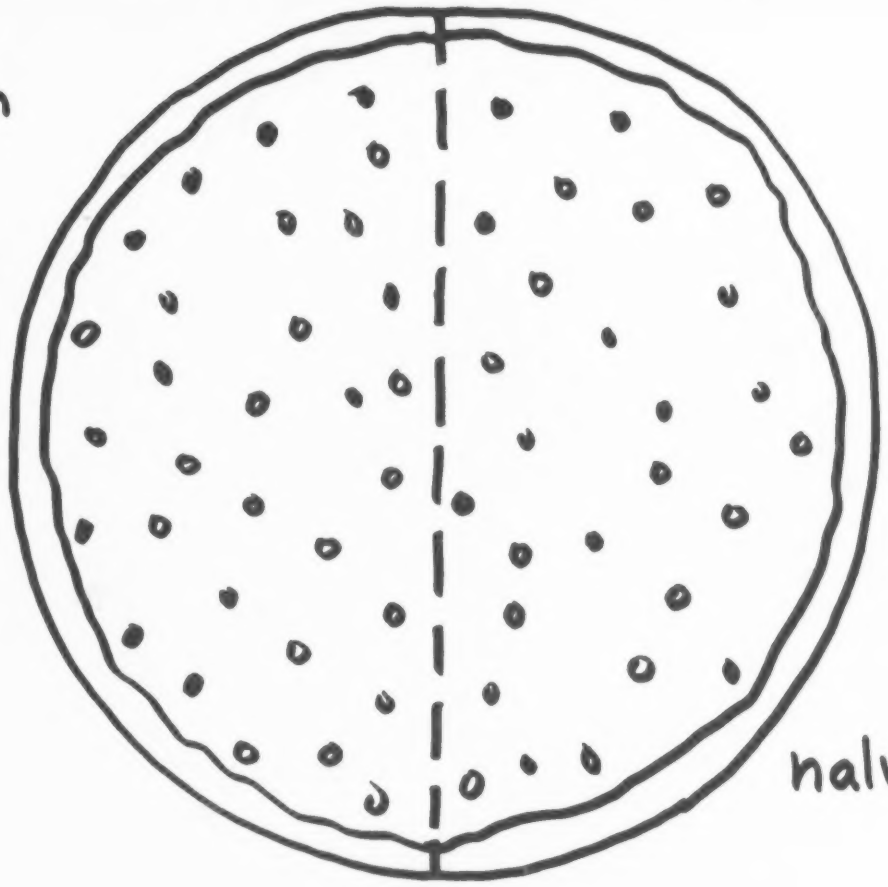
After the preceeding discussion, assign teams of children to choose an appropriate measuring tool (cm ruler or meter stick) and actually measure some distances. Discuss how one might measure a distance in km. If possible, take the children for a 1 km walk.

Let participants tell some measures they could take in their classrooms & schools.

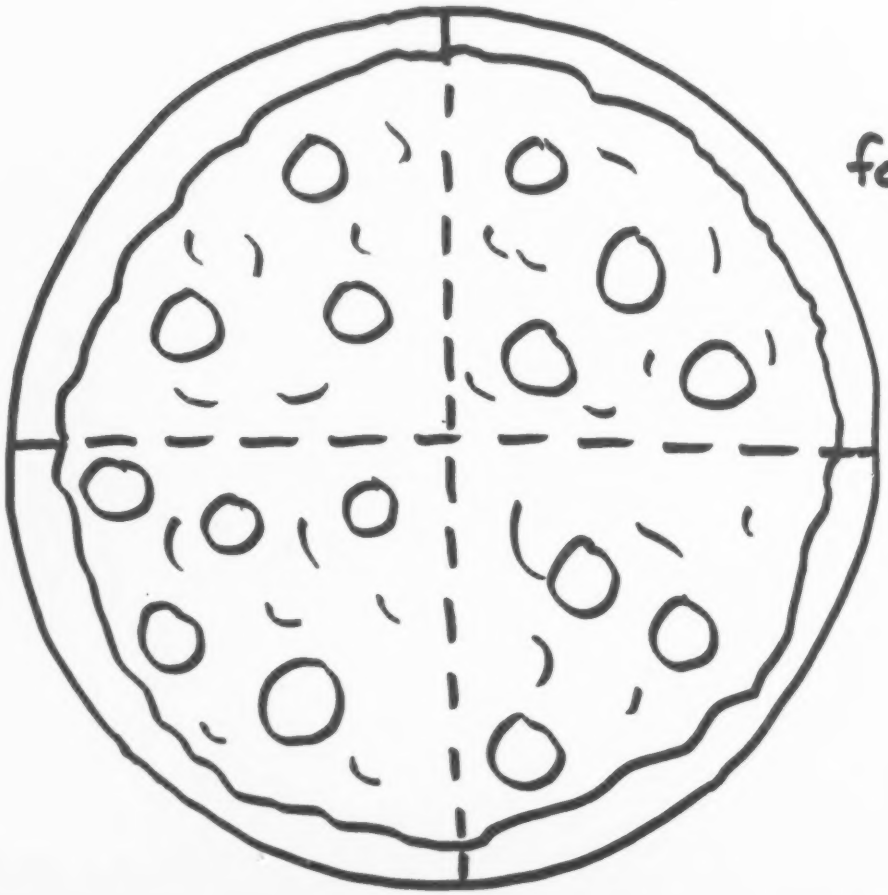
Groups measure assigned distances. If possible, they label each with a metric awareness sign.

Metric Awareness
Across the chalkboard
5m
measured by Bobby & Ken

Fraction Pizza Patterns

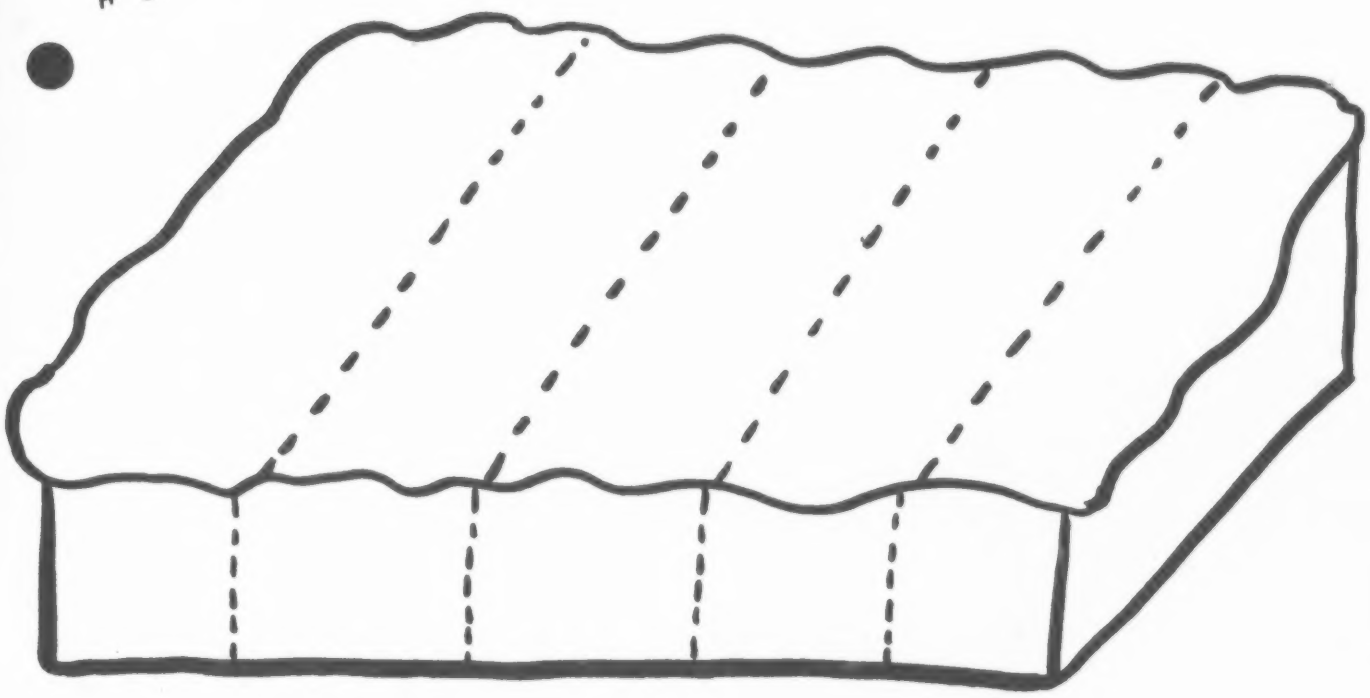


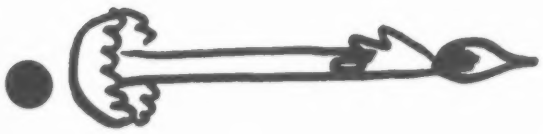
halves



fourths

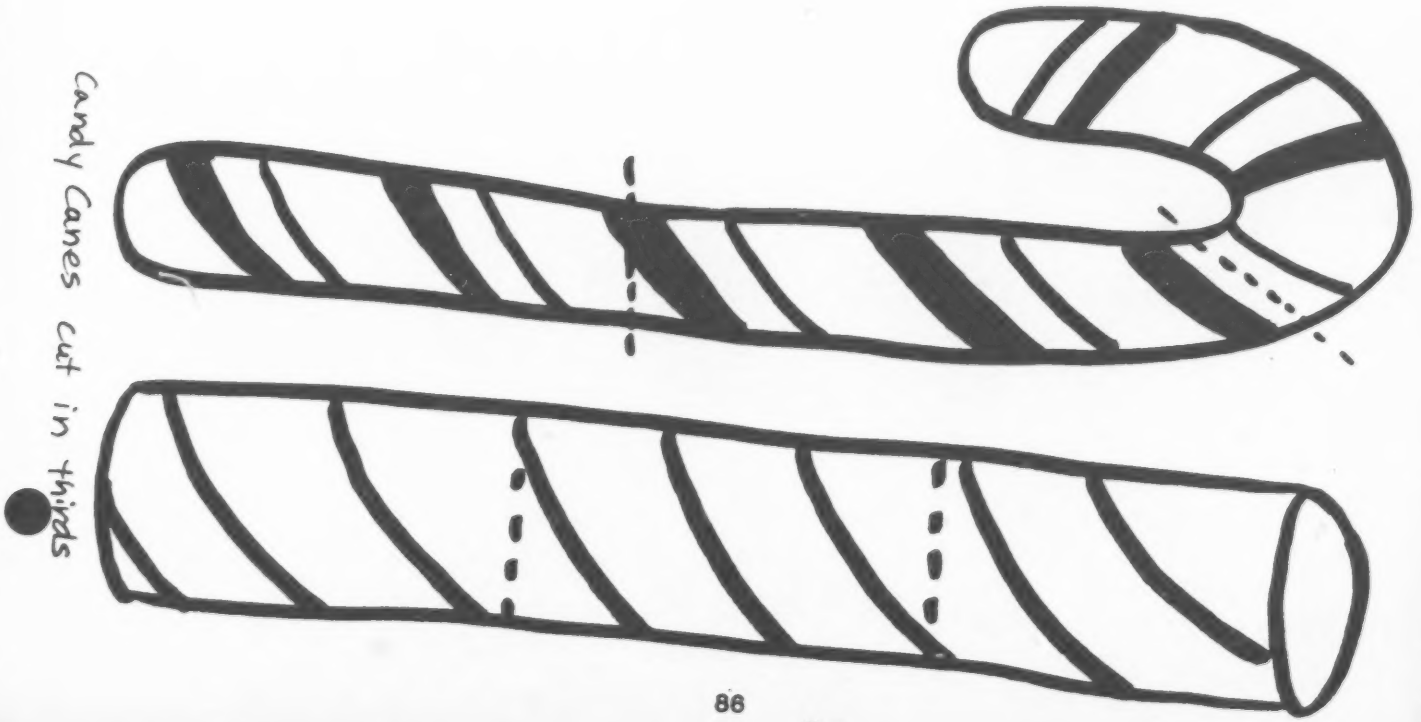
H-8

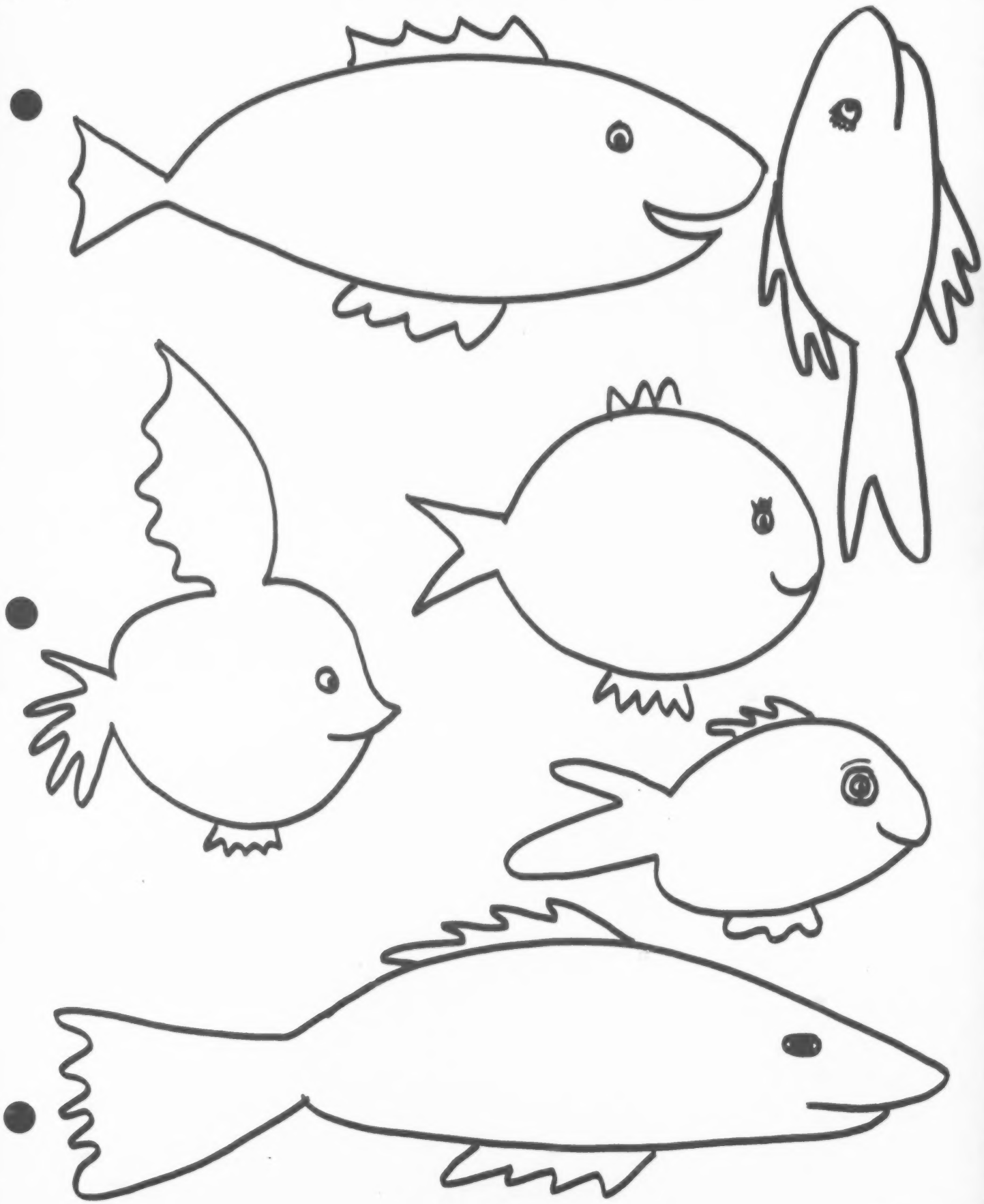




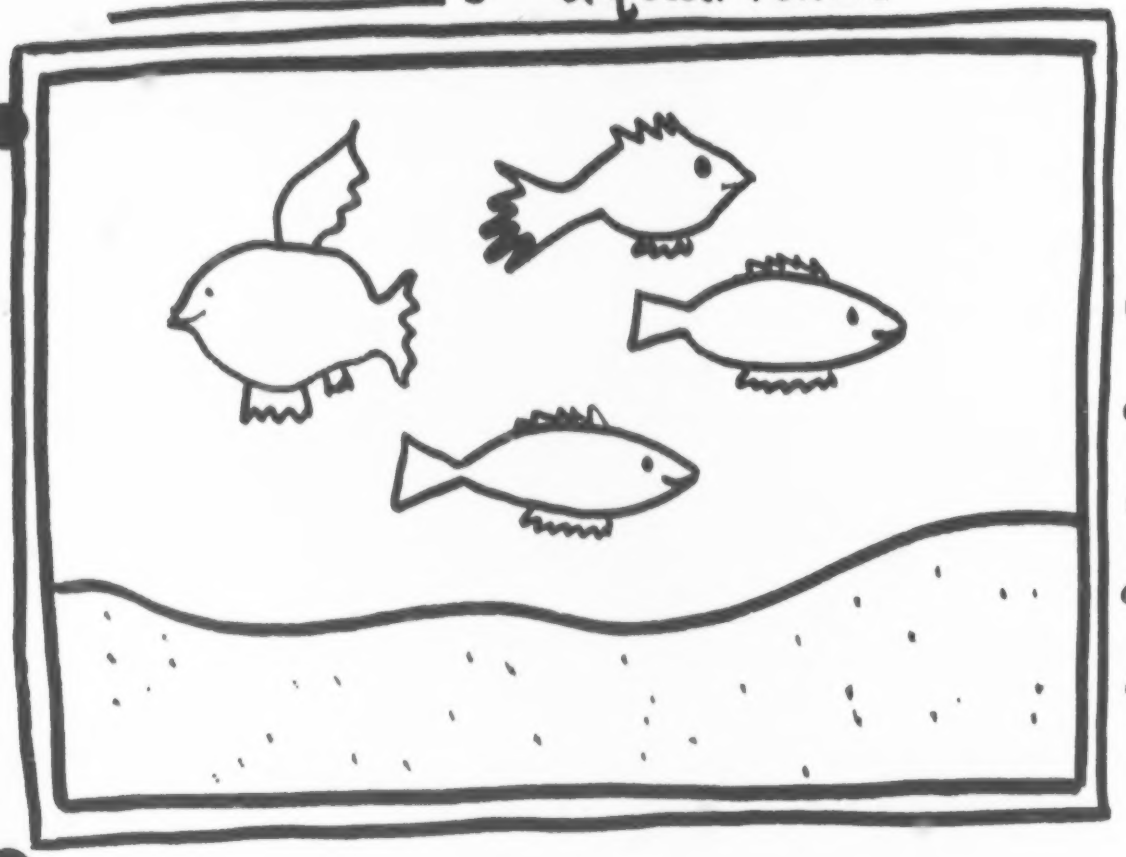
A cake cut in fifths. You could duplicate candles and have children put candles on 1/5, 3/5, etc. Or put candles on 2 sections and tell the fraction with and without candles.

Candy Canes cut in thirds

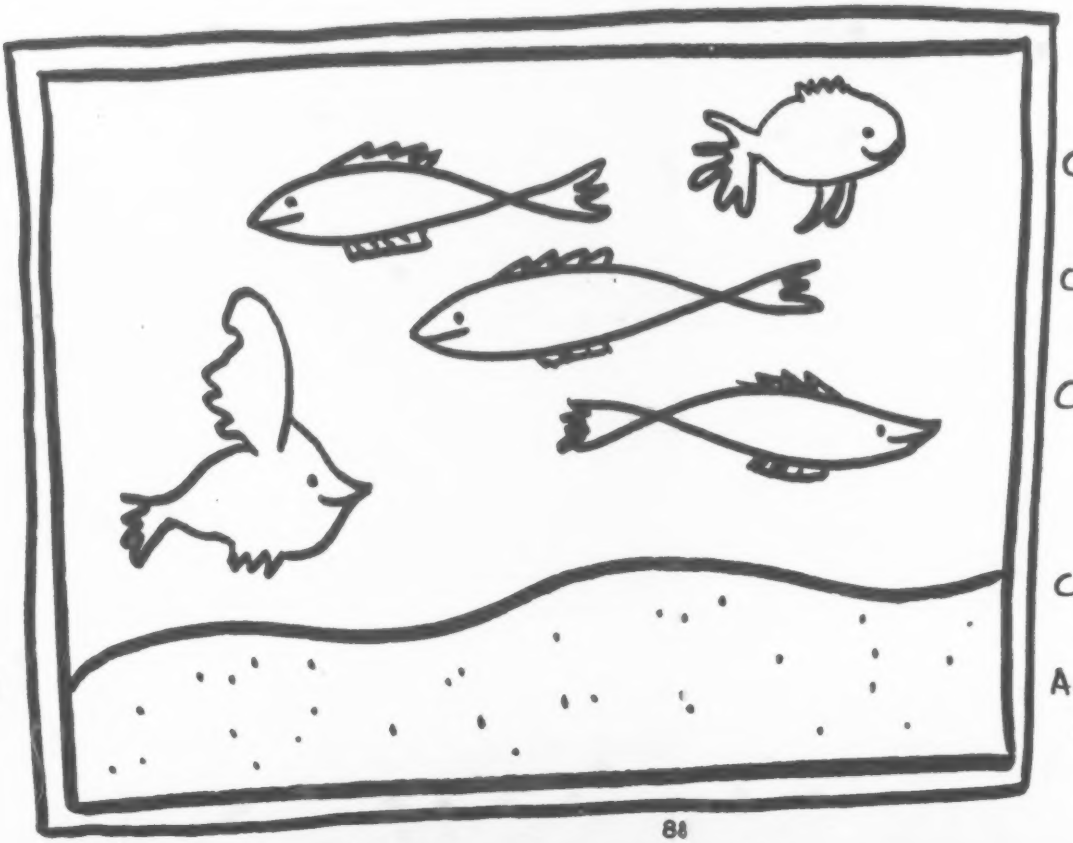




_____'s aquariums

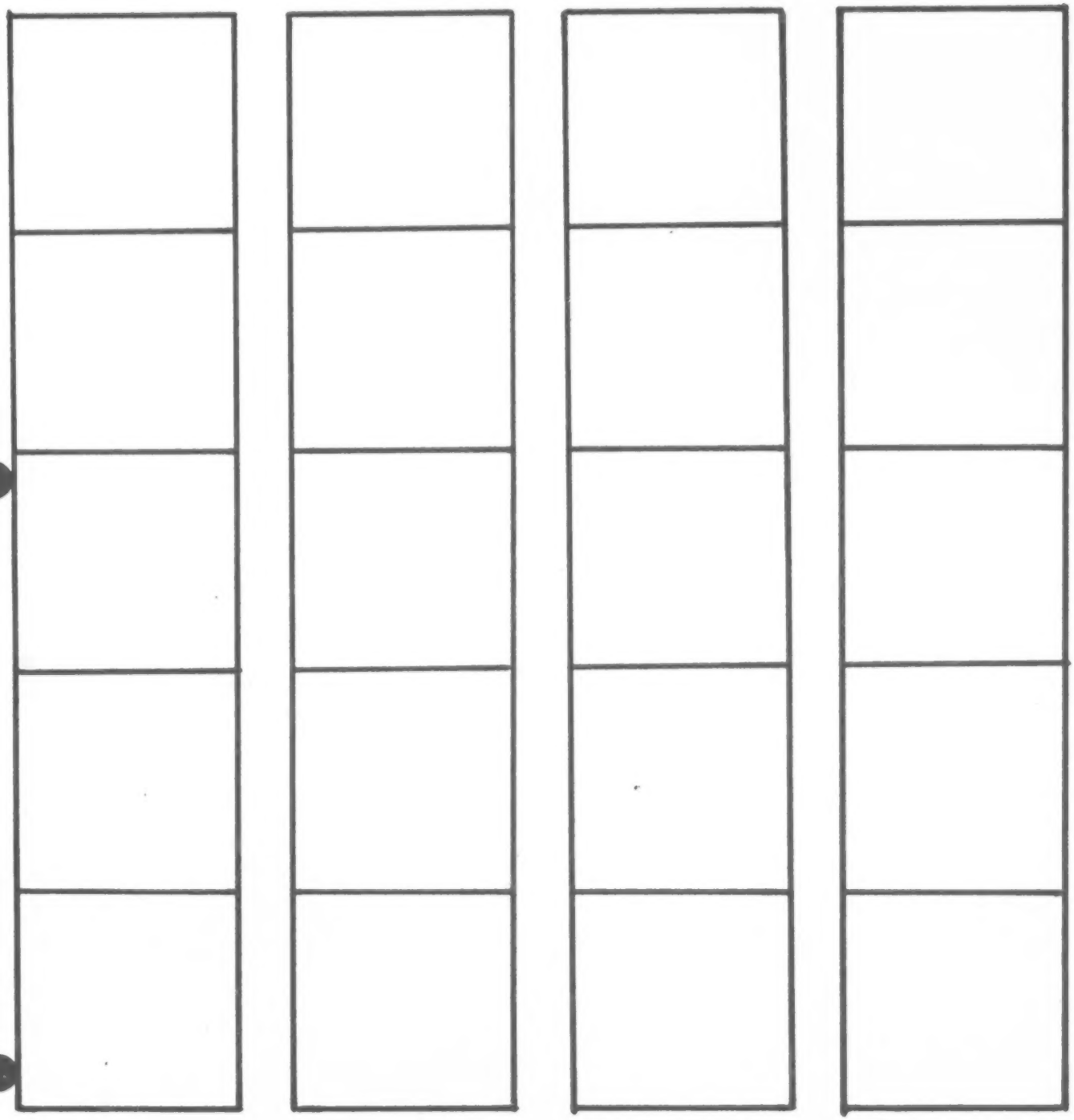


Color $\frac{1}{4}$ of the fish red.
Color $\frac{1}{2}$ of the fish green.
Color $\frac{1}{4}$ of the fish yellow.
Color the sand brown.
Add some pictures of plants.



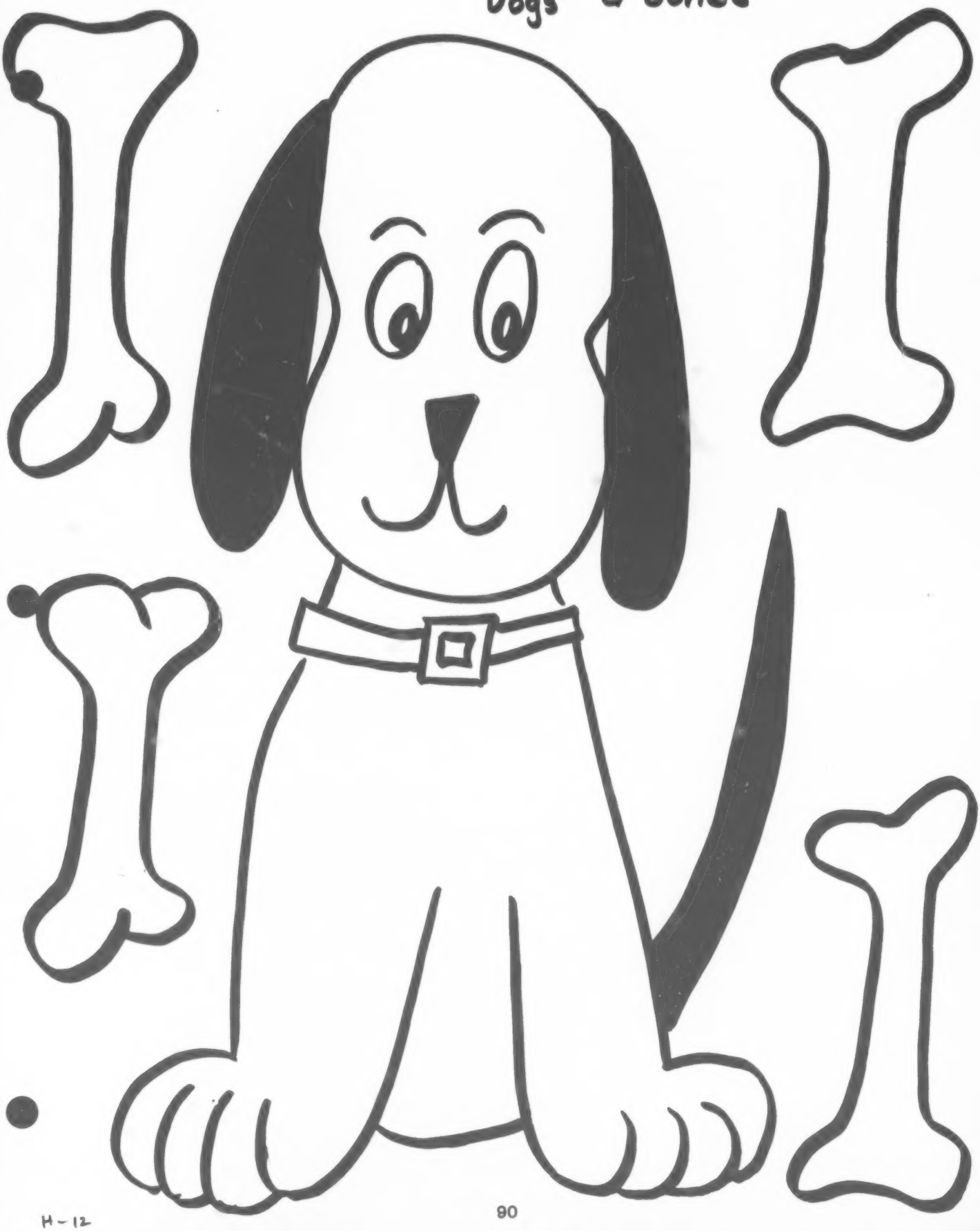
Color $\frac{2}{5}$ of the fish green.
Color $\frac{1}{5}$ of the fish orange.
Color $\frac{2}{5}$ of the fish a color you choose.
Color the sand grey.
Add some other things to the picture.

Graph Title



Category Name | Category Name | Category Name | Category Name

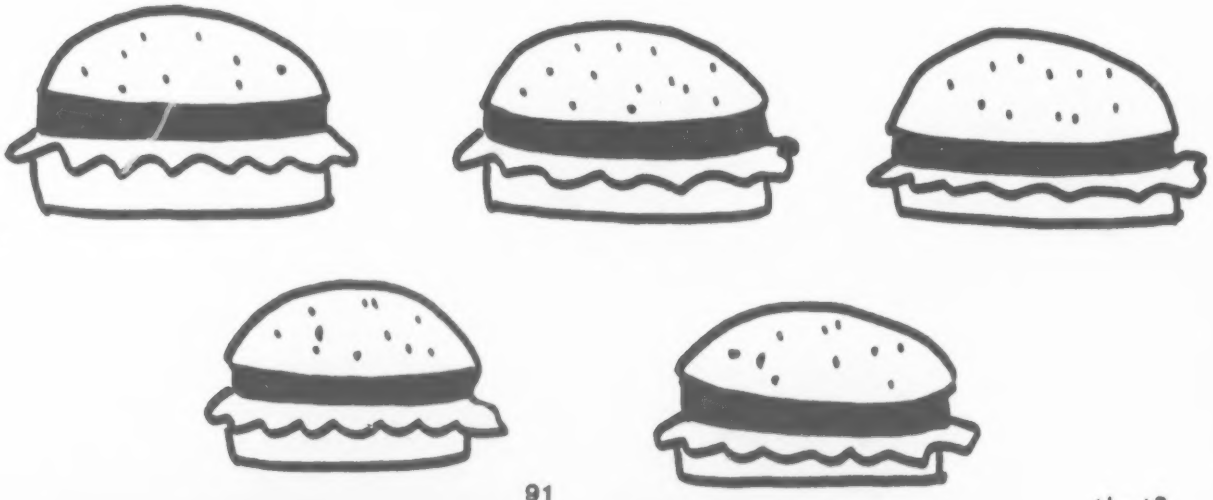
Dogs & Bones

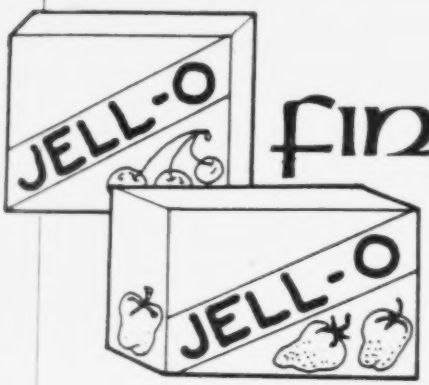


H-12

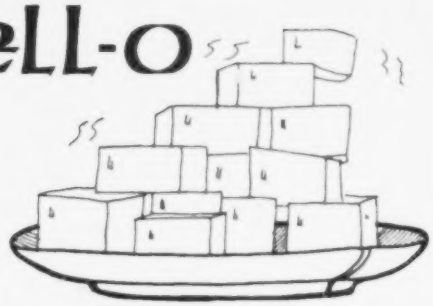
90

BEARS & HAMBURGERS





Finger Jell-O



4 pkgs. unflavored gelatin

500 ml hot water

2 small pkgs. flavored gelatin

500 ml cold water

What to do:

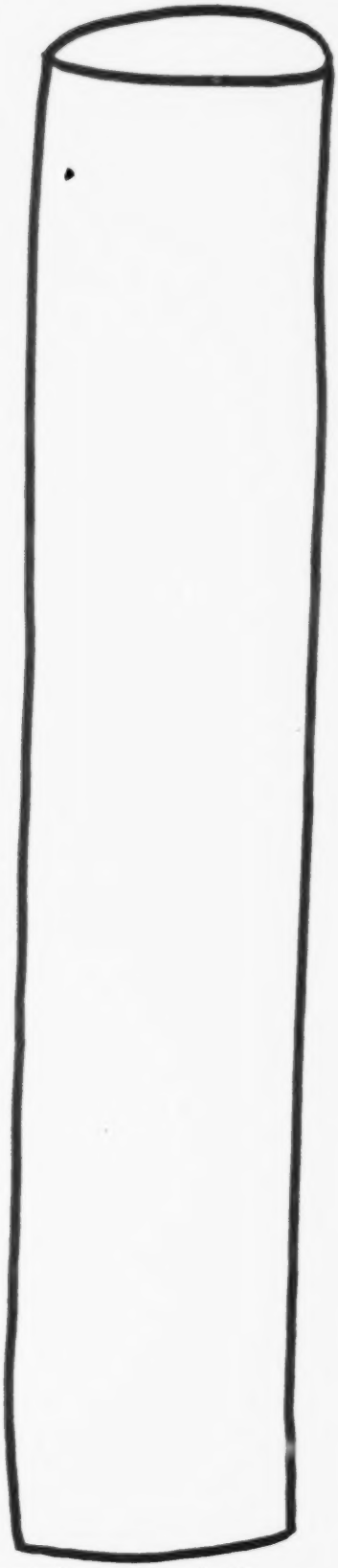
Stir unflavored gelatin into 100 ml hot water. Let soften about 15 minutes. Add flavored gelatin and 400 ml hot water. Stir until well dissolved. Add cold water. Very lightly grease a 22 x 30 cm pan or spray with a vegetable shortening. Pour gelatin in. Refrigerate until gelatin is firm.

From *Recipes for Learning* by Gail Lewis and Jean M. Shaw, ©1979 by Goodyear Publishing Company, Inc.

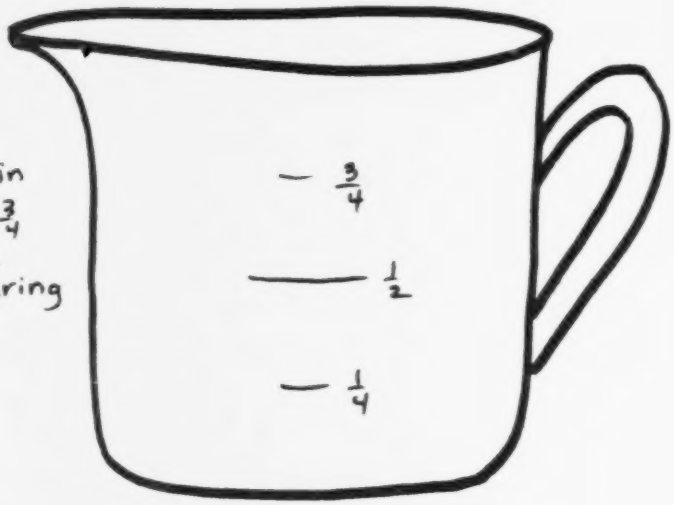
PARTLY FULL

name _____

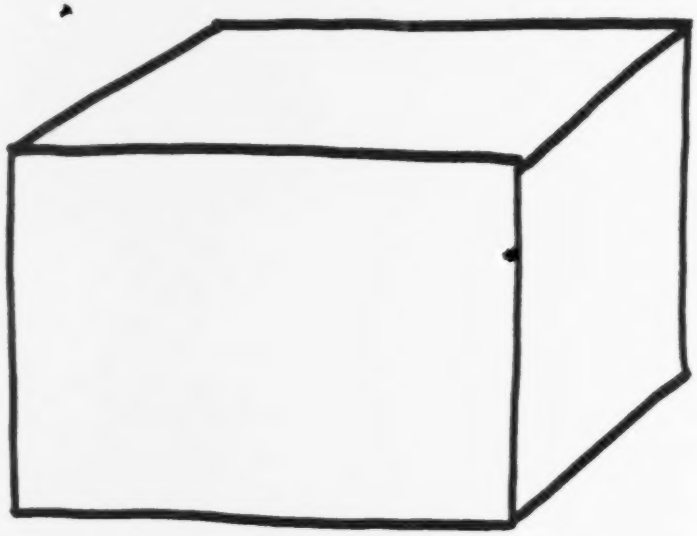
H - 15



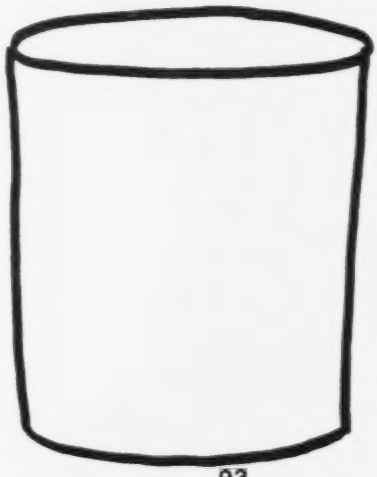
Make the tall container look 1/3 full.



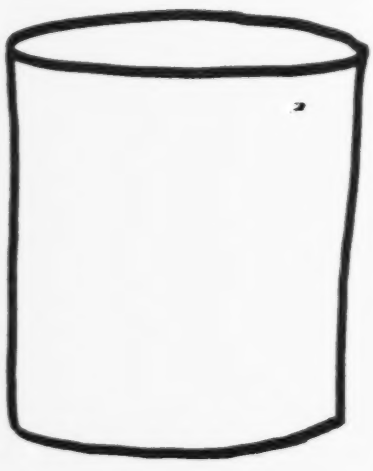
Color in about $\frac{3}{4}$ of the measuring cup.



Make the box look $\frac{1}{2}$ full of sand



Draw this glass about $\frac{1}{5}$ full of chocolate milk.

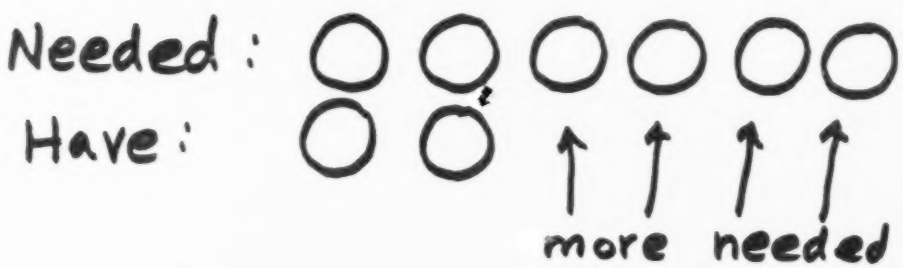


Draw this glass about $\frac{4}{5}$ full of chocolate milk.

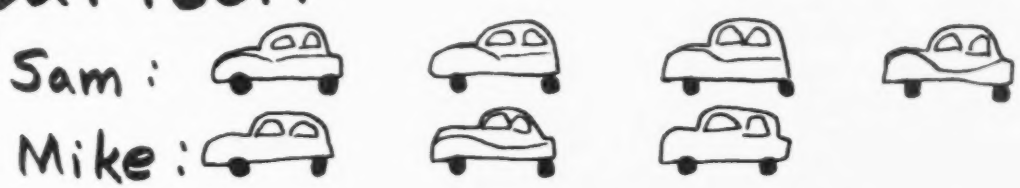
SUBTRACTION MODELS

Take-Away

More Needed



Comparison



Which is more? How many more?

Partitioning

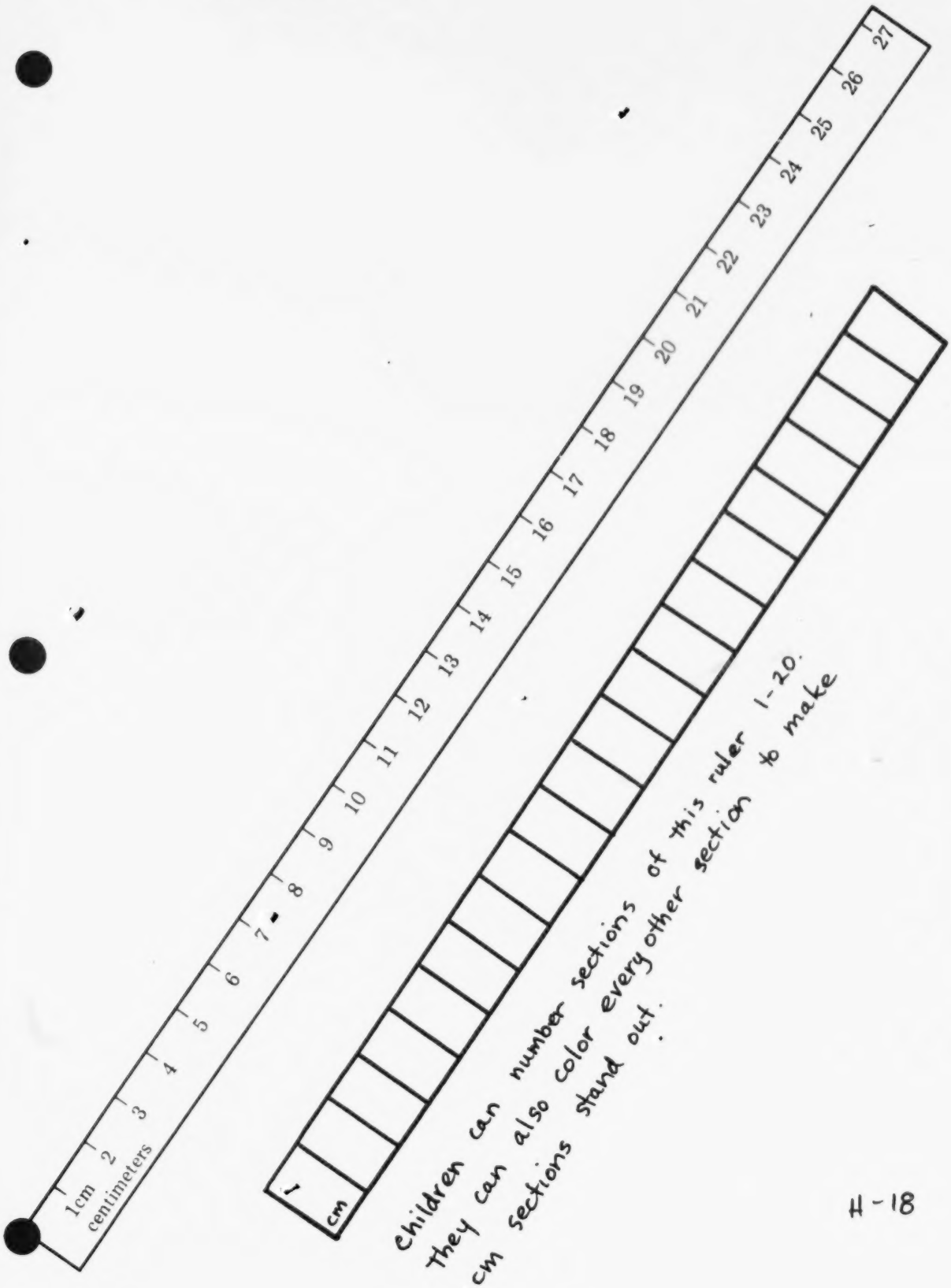
5 children
3 with jeans
How many without jeans?

Addition Chart

+	0	1	2	3	4	5	6	7	8	9
0	0	1	2	3	4	5	6	7	8	9
1	1	2	3	4	5	6	7	8	9	10
2	2	3	4	5	6	7	8	9	10	11
3	3	4	5	6	7	8	9	10	11	12
4	4	5	6	7	8	9	10	11	12	13
5	5	6	7	8	9	10	11	12	13	14
6	6	7	8	9	10	11	12	13	14	15
7	7	8	9	10	11	12	13	14	15	16
8	8	9	10	11	12	13	14	15	16	17
9	9	10	11	12	13	14	15	16	17	18

What patterns do you see?

CENTIMETER RULER



Children can number sections of this ruler 1-20.
They can also color every other section to make
cm sections stand out.

H-18

PROBLEM SOLVING

PROBLEM SOLVING

In this workshop session, participants will receive information on the importance of problem solving and general steps in problem solving. They will hear about many different kinds of problems appropriate for young children and will formulate and share examples of each kind. Participants will study problem solving strategies as presented by the leader and written materials from Arithmetic Teacher. Finally, workshop participants will share examples of the ways they might apply problem solving strategies in their own classrooms. The workshop leader should:

1. Discuss the importance of problem solving and a general method for problem solving.
2. Present examples of different kinds of problems and ask individuals to share more examples. Involve participants in group problem solving.
3. Overview problem solving materials from Arithmetic Teacher, then assign groups to study the materials and summarize these for everyone.
4. Present strategies and techniques for problem solving and solicit examples of each. Involve participants in more problem solving.
5. Prepare the following materials in advance:

   Transparencies P-1, P-2, P-0-3, P-6, P-7, P-8, P-10 through P-14.

   Handout sheets P-4, P-15.

   Handouts from Arithmetic Teacher — one per group or one per participant (P-9).

1. Importance of problem solving. In its position paper, Agenda for Action: Recommendations for School Mathematics of the 1980's, the National Council of Teachers of Mathematics set forth as its first recommendation:

   Problem solving must be the focus of school mathematics of the 1980's. (Transparency P-1.)

   Members of the NCTM, an active professional organization, admit that this recommendation is a challenge. They see problem solving as a process. They describe good problem solvers as persons with desire, enthusiasm, facility, and ability. Cultivating these traits can begin early — with primary grade teachers and assistants.

   To become effective problem solvers, children need lots of practice in "natural," motivating situations. They need to see the need for thought about a problem, planning how to solve the problem, carrying out the plan, and checking the solution. (Transparency P-2.) George Polya proposed these four steps for problem solving. The steps apply to problems encountered by adults as well as by children. Let's examine the four steps in some detail. (Transparency P-0-3.)

   Understanding the problem. Young children can act out problems or use manipulatives to demonstrate the facts given in a problem situation. They might draw a picture of the problem to increase their understanding of it. This step may be time-consuming but is essential. Children need time to "get to know" a problem before they can go on to solving it. They must clearly understand what is known and what is unknown in the problem.

   Planning how to solve the problem. Children can look for key words such as "altogether," "total," "how many more than" for clues to which operation(s) to use in the problem. A table or graph may help in seeing patterns in a problem. Setting up a number sentence may also help clarify what to do. Children may also guess at a value to solve the problem, try out their guess, and then decide how to adapt their guess to make it correct. Instead of working a problem with difficult numbers, children can be taught to simplify the numbers so they can see what to do in the problem. Instead of "Jack has $1.88 Bill has $5.97. How much more money does Bill have?," children might think "Jack has $2; Bill has $6, so I know that Bill has $4 more; I'm comparing so I can subtract the smaller from the larger number."

   Carrying out the plan — doing the arithmetic. Ordinary computational skills are important in this step. Children should be encouraged to do their best work. At any level, children may use calculators to help with arithmetic operations.

Checking the solution. Children should be taught to routinely look at their answers and see whether the answers are sensible and reasonable. Next they can check the details of their arithmetic and make sure their answer is correctively labeled with units such as dollars, pounds, or another appropriate word. To extend children's understanding of problem solving, children might also talk about various ways to solve problems, and discuss the processes they used in their solutions. Research shows that time spent reconsidering the thinking involved in problem solving helps children to become better problem solvers.

2. Different kinds of problems. Children must be exposed to many different kinds of problems. One goal is that children will recognize that the process of problem solving is of primary importance — meaningless answers detract from the goal of understanding. Another goal of presenting various problems is to prepare children for many different kinds of problems they will face in life. Some suggestions for different kinds of problems follows:
   - Routine and unusual problems. Routine problems involve the application of mathematical procedures in the same way the procedures were learned. Typically, these routine problems follow computation sections in texts and workbooks. Typically also, these routine problems require little thought! Solutions to unusual problems, on the other hand, are not as obvious. Children need to apply the problem solving steps just outlined. For some examples, use Handout P-4 and have groups of participants try to solve at least one problem. Let them briefly share some strategies they used to solve their problems. (Answers are on P-5.)
   - Open-ended problems. These problems involve decision making and have several correct answers. They are similar to many problems children encounter in everyday life. Which problems on Handout P-4 are open-ended? (The menu and dart problems are open-ended; "Change" and square problems also have several solutions.) Transparency P-6 shows more open-ended problems. What other open-ended problems can you suggest?
   - Problems with too much or too little information. Children need exposure to these kinds of problems for practice in picking out pertinent information. Like open-ended problems, these problems are frequently encountered in everyday life. They can be easily posed to children based on classroom situations. Some examples are on Transparency P-7. Think of some more classroom situations where too much or too little information might be given to children.
   - Problems where a number sentence or equation is helpful. Some textbook authors present the writing and solving of number sentences as the one approach to problem solving. Research indicates that while this approach is useful, it is not so useful as to be taught exclusively. In working with open sentences, children can practice writing and discussing many problems to fit a given open sentence. This will help to emphasize the idea that a sentence or equation represents a whole class of problems. (Transparency P-8.) Children must remember to check the numbers they substitute in open sentences to make sure the numbers make the sentence true. What other problems can you make for the equations on the transparency?
3. The National Council of Teachers of Mathematics published several different problem solving strategies in Arithmetic Teacher for the 1984-1985 school year. (Handouts P-9.) Each month's issue featured a different type problem solving strategy. We will look at some of the Arithmetic Teacher materials. As you look, try to summarize the strategy and pick problems appropriate for the children you teach. Make some other similar problems that you could use in your classroom (Break participants into 8 groups to further explore the Arithmetic Teacher materials. Invite them to briefly share results.)
4. What are some other effective approaches to problem solving? In addition to the approaches and kinds of problems already suggested, other guidelines and techniques are pertinent: (Transparency P-10.)

   Allow time and opportunity for group work on problem solving. Many heads are better than one! Even the youngest elementary children can learn to work together in small groups to solve problems. Studies have shown that groups do

solve more problems than individuals, though the group process takes more time. As children work in small groups, they can discuss problems, share ideas, and verify solutions. Often groups will find several viable approaches to problems. When two to four children work together, they usually all end up learning from the others.

Encourage children to make up and solve their own problems. This technique creates much interest and motivation. Children use their own language and thus make meaningful problems. You might let groups of children use interesting pictures and create problems based on the pictures. Let's try this technique. (Use a current advertisement that would be of interest or another interesting picture and invite participants to make several problems based on the pictures.) Ask children to make their own drawings and write or dictate problems based on the drawings. Invite children to make word problems related to stories they are studying in reading, science, or social studies. Children can share their problems with the class by acting them out, then discussing them or by placing them on a bulletin board for others to read, take down, and solve. Groups of children may also make graphs and invite others to answer questions based on their graphs. (Transparency P-12.) (Have participants model this behavior.)

Provide a place, time, and motivation for individual efforts in problem solving. Educators can create a little spot in their classrooms for problem solving. This might be (Transparency P-13) a Puzzle Corner with several problems and a space to sign "I've solved it!" You might fix a bulletin board for child-made problems and assign children to copy, solve, and check 3 or 4 problems as their daily work. A learning center might feature geometry or number puzzles along with other learning activities. Both adults and children can contribute to a puzzle file that children are free to use. The file should have answers available — on separate cards or on the backs of problems. What other techniques can you suggest for promoting individual, self-directed efforts in problem solving?

Problem solving involves many different steps. At times concentrate on a single step of problem solving. For example, you may read problems to children and ask them only to decide which operation they would use and why. You might also present some problems where children simply decide "What do we know?"and "What do we need to know?" Still another time you might use several word problems and concentrate on the equation or number sentence-writing step. Another day you may want to take some problems, state an answer for each, and ask the children to check or verify the answer to see if it is correct.

Teachers and teaching assistants must gather and share materials and appropriate problems. Many resources for problem solving are available. Children can be stimulated by use of interesting, challenging, and appropriate materials. Professionals may want to: (Transparency P-14.)

* Collect potential problems from magazines, puzzle books, newspapers, catalogs, and other sources.
* Write their own problems based on seasonal or current interests and events.
* Attend problem-solving sessions at professional meetings. Share materials and ideas with colleagues.
* Be aware of and use problem-solving in other curricular areas.
* Use brief, orally presented problems as an effective means of filling time while children are waiting to go to recess or lunch.

Ask participants to share other approaches or techniques they have found to be helpful or effective. Tell participants that a future session or evaluation will include special techniques for assessing children's problem-solving abilities. As a "Wrapup" activity, involve participants in more problem solving. Ask them to work with a partner on at least one problem from handout sheet P-15, then work at least one more problem independently. Show some answers and strategies for the problems (Answers are on P-5.)

FIRST RECOMMENDATION

Agenda for Action: Recommendations for School Mathematics of the 1980's

National Council of Teachers of Mathematics

"Problem solving must be the focus of school mathematics of the 1980's."

George Polya's 4 STEPS in PROBLEM SOLVING

- Understand the problem.
- Plan how to solve it.
- Do the arithmetic.
- Check the solution.

- use representations or drawings
- act it out
- see what's known & unknown

- "clue words", charts
- make a guess
- solve an easier problem

- do your best, careful work
- use calculators

- sensible?
- correct?
- units correct?
- look back

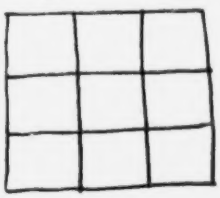
• PROBLEMS •

Find 21 coins to make change for $1.00.

Choose a drink, a sandwich, and something else for under $2.25:

milk -- 35¢, 50¢
soda -- 35¢, 50¢, 60¢
superburger -- $1.50
hamburger -- $1.25
cheese sandwich -- 75¢
tuna sandwich -- $1.00
french fries -- 50¢
grits -- 35¢
onion rings -- 75¢

Place the numbers 1-9 in the squares so that the sum in any direction, including diagonals, is 15.

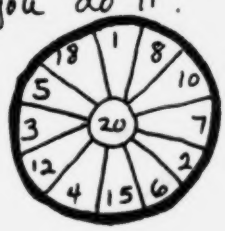


Put signs in the problems to make a true sentence:

8 8 2 5 = 95

1 0 0 1 7 = 83

"Hit" the target with 4 scores to give you 25 points. How many different ways can you do it?



Find 11 coins to make change for $1.00

Complete the patterns:

20, 15, 17, 12, 14, ___, ___, ___, ___

2, 4, 5, 10, 11, 22, ___, ___, ___, ___

1, 2, 4, 8, 16, ___, ___, ___, ___

ANSWER SHEET TO P-4 AND P-15

ANSWERS TO P-4

1. Two possibilities are:

10 pennies	= $ .10	13 nickels	= $ .65
7 dimes	.70	3 dimes	.30
4. nickels	.20	5 pennies	.05
21 coins	$1.00	21 coins	$1.00

   Can you find others?

2. Menu — There are many possible answers. One strategy to staying under $2.25 might be to choose a sandwich, the most expensive item, first, then choose other items.

3. A square where each row, diagonal, and column sum is the same is called a magic square. Here's one way to solve:

4	9	2
3	5	7
8	1	6

   Can you find other combinations?

4. 88 +2 + 5 = 95

   100 - 17 = 83

5. There are many possible answers for the target. Here are a few combinations of which the sum is 25:

20, 1, 1, 3	12, 4, 6, 3	10, 5, 2, 8
10, 6, 4, 5	15, 2, 2, 6	12, 5, 3, 5

6. Eleven coins that equal a dollar might be:

7 dimes	= $ .70	2 quarters	= $ .50
6 nickels	.30	8 nickels	.40
11 coins	$1.00	1 dime	.10
		11 coins	$1.00

   Check other combinations too!

7. 20, 15, 17, 12, 14, 9, 7, 2, 4

   2, 4, 5, 10, 11, 22, 23, 46, 47, 94

   1, 2, 4, 8, 16, 32, 64, 128, 256

ANSWERS TO P-15 (and some questions)

1. One approach is to start guessing and making a chart. Your work might look like this. You can see that if the carpenter made 5 of each, the number of legs is too small. So vary the numbers as shown in the 5th and 6th lines. The 6th line gives the correct total — from 5 stools and 4 tables. Are there other answers?

Stools	Stool Legs	Tables	Table Legs	Total Legs
1	3	1	4	7
2	6	2	8	14
3	9	3	12	21
5	15	5	20	35
4	12	5	20	32
5	15	4	16	31

2. Trace various paths. There are hundreds of possibilities.
3. Uh oh! This problem has no solution. Sometimes that happens. The sum of any two odd numbers is even; 3 odd numbers' sum is always odd; 4 odd numbers always add up to an even number; 5 odd numbers' total is always odd! Try lots of combinations of 5 to see that the sum is always odd. Or look at the algebraic proof:

   An even number is represented 2n.

   An odd number is represented (2n+1)

   The sum of 5 odd numbers is represented:

   $(2a+1) + (2b+1) + (2c+1) + (2d+1) \pm (2e+1)$

   The expression can be simplified:

   $2a + 2b + 2c + 2d + 2e + 5$ or

   $\underbrace{2(a + b + c + d + e)}_{\text{even}} + 5$

   This part is even and 5 is odd.

   Therefore, the sum is odd — always!

Probably children should only be asked to encounter math problems with no solutions after much exposure to problems with answers.

OPEN-ENDED PROBLEMS

- You have $3.00 to spend. What will you buy?

- How many different ways can you make change for $1.00 ? Show me.

- Your parent says you can watch TV for only 2 hours on Saturday. What will you watch? (Use guide)

- What numbers fit? $\triangle - 2 = \square$

Problems with TOO MUCH or TOO LITTLE data.

- Some children order pictures. How much money does the teacher collect?
- (Use children's names & situations.) Andy has 4 brothers and a sister. Billy has 1 sister. Charlie has 2 brothers and 1 sister. How many children in Andy's and Charlie's families?
- Track records for 8th grade boys.

100 m dash

	record est	time
Charlie B.	1981	12 sec.
Kevin G.	1982	11 sec.
Billy S.	1984	13 sec.
Joel W.	1985	10 sec.

How long ago were records established?

Who ran fastest? By how much?

How old was Charlie B.?

MANY PROBLEMS for the SAME OPEN SENTENCE

$\square + 2 = 5$

Jody has some dogs. She gets 2 more. Now she has 5. How many dogs did she start with?

There are 5 cars. Some are red; 2 are blue. How many are red?

$2 \times \square = 10$

2 baskets. Same number of eggs in each. 10 eggs in all. How many eggs in each?

2 people. Each spends same amount of money. They spend $10 in all. How much does each spend?

P-9a

Problem Solving Tips for Teachers

By **Phares G. O'Daffer**
Illinois State University, Normal, IL 61761

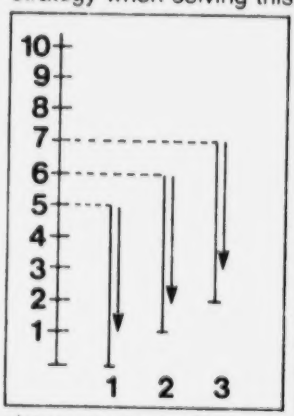
Strategy Spotlight

Draw a Picture

Draw a picture is a strategy that is very helpful in solving certain problems. Consider the following problem to see how this strategy is used.

Problem: A sleepy koala bear wants to climb to the top of a fruit tree that is 10-m tall. Each day the bear climbs up 5-m, but at night, while asleep, slides back 4-m. At this rate, how many days will it take the bear to reach the top of the tree?

It is natural for children to choose the draw-a-picture strategy when solving this problem. Note that the picture can be drawn freehand and need not be a professional rendition. However, it must be drawn and labeled so that the conditions of the problem are clearly shown. If the picture shown here is continued, the student will see directly from the picture that the bear will reach the top of the tree on the sixth day.



However, the draw-a-picture strategy often sets the stage for a more efficient solution to the problem by helping children see that other strategies can be used. For example, encourage students to look for a pattern in the picture. They might observe that at the end of day 1, the bear is at the 1-m mark and at the end of day 2, the 2-m mark. So at the end of day 5, the bear would be at the 5-m mark. Thus on day 6, the bear would climb 5 m and reach the top. Some students may start at 10 and work backward by counting back 5 to see that the bear will reach the top on the day that it begins climbing at the 5-m mark. Help students understand the role of the draw-a-picture strategy and compliment them when they use it to get started on a solution to a problem.

In the September 1984 "Problem Solving" section, a plan for guiding a problem-solving experience was given. If you use that plan with the koala bear problem presented here, try these questions:

- *Questions for understanding*
  1. What is the bear doing?
  2. How tall is the tree?
  3. How far did the bear climb during the day?
  4. How far did it slip back at night?
  5. What does "reach the top" mean?

- *Questions (hints) for solving*
  1. Would it help to draw a picture?
  2. The answer isn't 10 days. Why not?
  3. Where is the bear at the end of day 1? Day 2? Day 3? Do you see a pattern?

Classroom Climate

Having children work in groups of three or four helps develop a healthy classroom climate. The establishment of ground rules at the beginning is very useful. Try these:

1. Be involved in your group. (Share your ideas. Listen only to *your* group.)
2. Be or use a source of help. (Help anyone who asks. Ask for group help if needed.)
3. Make decisions in your group. (Ask your teacher only when your group can't figure a problem out. The group should agree before giving an answer to a problem.)

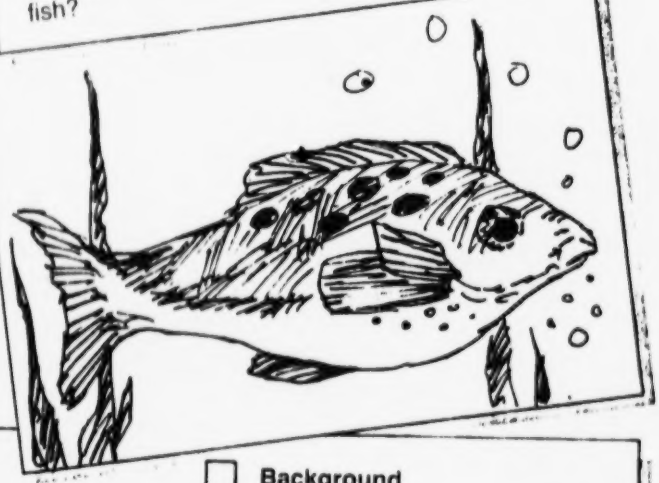
Take a Look

The 1983 NCTM Yearbook, *The Agenda in Action*, (pp. 9–87) contains ideas for problem-solving activities in preschool and primary grades as well as additional techniques for upper grades.

Problem Corner

Students in middle school might like to solve this one! Drawing a picture is helpful.

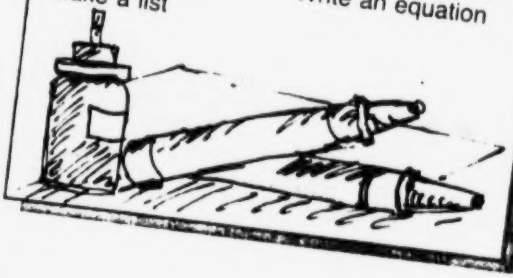
The head of a fish is ⅓ as long as its body. The tail of the fish is as long as its head and its body combined. The total length of the fish is 48 cm. How long is each part of the fish?



Developing Problem-solving Skills

Problem solving is a process that requires skills in *understanding the question, collecting data, planning what to do, finding the answer,* and *checking* to see if the answer makes sense. To help develop skills in planning what to do, make a **problem-solving strategies** poster. As you focus on new strategies, add them to the poster and encourage students to refer to them as they plan a solution to a problem. Here are some useful strategies:

Choose operation(s)
Guess and check
Draw a picture
Make a table
Make a list
Work backward
Solve a simpler problem
Look for a pattern
Write an equation



Background

Here are two definitions of a *problem*. Which one do you like best?

1. A real problem for an individual is a question that presents a challenge, that is, cannot be solved by a routine procedure already known to the individual, and whose challenge is accepted by the individual.
2. In the elementary school a problem is a situation given to a student, usually by a teacher, where the student must use one or more solution strategies to answer a question or make a decision.

How would you change these definitions?

P-9c

Problem Solving Tips for Teachers

By **Phares G. O'Daffer**
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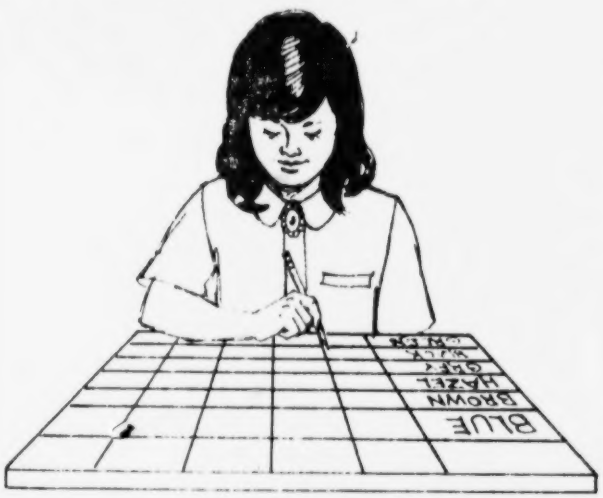
Strategy Spotlight

Make a Table

Make a table is a very useful strategy. Consider the following problem to see how this stragegy might be used:

PROBLEM: Carlotta read that 2 out of every 5 people have blue eyes. She decided to use this idea to predict how many pupils in her class of 30 would have blue eyes. How many do you think she predicted?

Although this problem can be solved using the operations of division and multiplication, it can also be solved by making a table.



- Use data in the problem to begin a table. The table should be labeled so it will be useful. Write numbers in sequence so that the patterns can more easily be observed. Check the entries in the table to see if they accurately show the relationships given in the problem.

Blue eyes	**2**					
Others	**3**					
Total	**5**	**10**	**15**	**20**	**25**	**30**

- Use these entries to extend the table to give new information. The relationship 2 to 5 is the key to extending the table.

Blue eyes	**2**	**4**	**6**	**8**	**10**	**12**
Others	**3**	**6**	**9**	**12**	**15**	**18**
Total	**5**	**10**	**15**	**20**	**25**	**30**

- Use the information in the table to solve the problem. When the total number of pupils is 30, about 12 should have blue eyes.

Blue eyes	**2**	**4**	**6**	**8**	**10**	**12**
Others	**3**	**6**	**9**	**12**	**15**	**18**
Total	**5**	**10**	**15**	**20**	**25**	**30**

Use this column

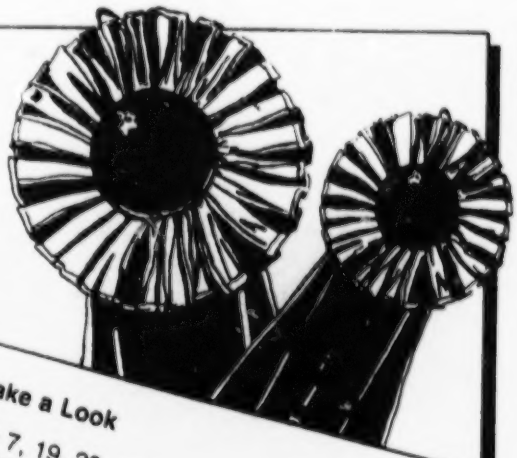
The **make-a-table** strategy is useful not only for finding the solutions for certain problems directly but often for organizing data when one employs other strategies, such as **guess and check** and **look for a pattern.** Include the **make-a-table** strategy on your classrooms' **Problem-solving Strategies** poster or bulletin board, and compliment students when they use it effectively in solving a problem.

P-9a.

Classroom Climate

The teacher's **enthusiasm** and what the teacher **rewards** play a major role in developing a special "problem-solving spirit" in the classroom. A nonverbal acknowledgment or a special comment that rewards the following will work wonders!

- Willingness to try—to take a risk
- Perseverance—to "stick to it"
- Self-reliance—to trust one's own ability
- Thinking—to think nonalgorithmically



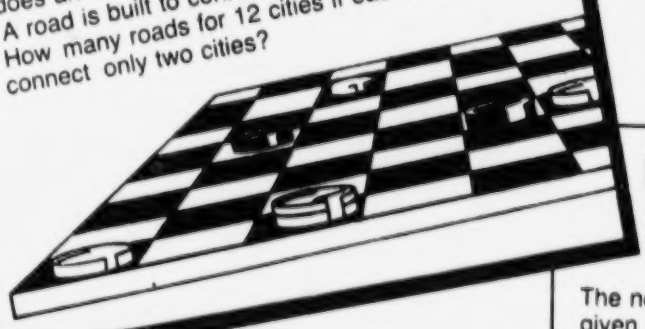
Take a Look

Chapters 7, 19, 20, and 21 of *Mathematics for the Middle Grades (5–9)*, the 1982 Yearbook of the NCTM, contain useful problem-solving ideas. NCTM's booklets *IDEAS from the Arithmetic Teacher (Grades 1–4 Primary, 1982; Grades 4–6 Intermediate, 1979; Grades 6–8 Middle School, 1982)* also have many good problems.

Problem Corner

Students in junior high school might like these! Making a table helps in their solution.

- How many squares (of all different sizes) does an ordinary checkerboard have?
- A road is built to connect each pair of cities. How many roads for 12 cities if each road can connect only two cities?



Give It a Try!

Use detective or mystery stories as an aid to help students problem-solving skills!

The need to understand the situation, consider given data, collect additional data, discard irrelevant data, analyze and draw conclusions from the data, devise a plan for solving, and finally, to solve and verify the reasonableness of the solution are common to solving both mysteries and math problems. Here are some references to help you begin.

Crouse, Richard, and Denise Bassett. "Detective Stories: An Aid for Mathematics and Reading." *Mathematics Teacher* 68 (November 1975):598–600.

Ripley, Austin. *Minute Mysteries*. New York: Harper & Row, 1976.

Scalzitti, Joyce. "Wanted Dead or Alive: Problem-solving Skills." In *Mathematics for the Middle Grades (5–9)*, 1982 Yearbook of the National Council of Teachers of Mathematics, edited by Linda Silvey, pp. 182–87. Reston, Va.: The Council, 1982.

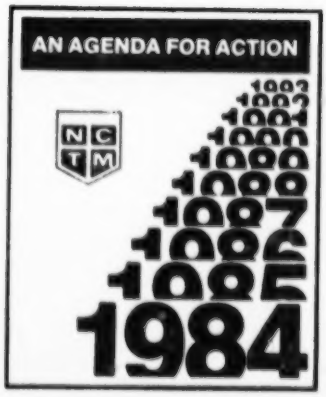
Sobol, Donald. *Two-Minute Mysteries*. New York: Scholastic Book Services, 1969.

Developing Problem-solving Skills

Students develop skill in problem solving by acquiring abilities to *understand the problem and question, deal with the data, plan what to do, find the answer,* and *check* to see if the answer makes sense. To help children deal with data, vary the sources of data. Assign problems with—

- data within the statement of the problem;
- data in a graph or table on the same page as the problem;
- data on a page different from the problem;
- data from an outside source (reference book, menu, map, tax table, computer data bank, time schedule, newspaper, videotape, etc.);
- data generated in an activity or experiment (e.g., a poll, a pulse count, or an event tally).

p. 9c



Problem Solving Tips for Teachers

By **Phares G. O'Daffer**
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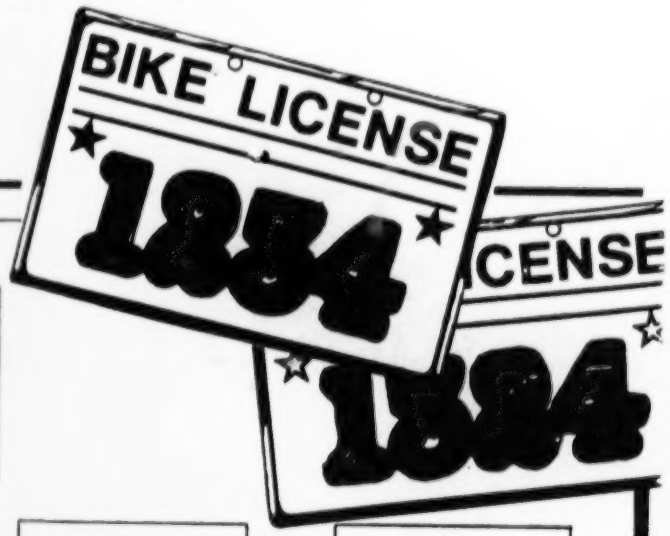


Strategy Spotlight

Make an Organized List

Make an organized list is a strategy that facilitates a systematic approach to certain types of problems. Consider the following problem to see how this strategy might be used.

Problem: Ann, Beth, Cathy, Dee, and Evita were playing in matches to see who would play position 1, 2, 3, 4, or 5 on the girls' tennis team. Each girl played each of the other girls once. How many matches were played?

Children often use the *make-an-organized-list* strategy when solving this problem. First, a child might list all the matches that person *A* plays. Then the child would list all the matches played by persons *B*, *C*, *D*, and *E*. Note that once match *AB* is listed, match *BA* involves the same players and is not a different match.

AB	AC	AD	AE
BC	BD	BE	
CD	CE		
DE			

As students begin to use a list to help them solve problems, emphasize the importance of using a *systematic* approach when making the list. For example, to solve the problem "How many different bike license plates can you make with the numbers 1, 2, 3, and 4?", it helps to list first all possibilities that start with 1, as shown:

1	2	3	4
1	3	2	4
1	4	2	3

1	2	4	3
1	3	4	2
1	4	3	2

In a similar manner, the six possibilities that start each of the other numbers can be listed, for a total of twenty-four licenses.

The *make-an-organized list* strategy is often used to solve problems that involve the ideas of permutations (e.g., the license problem), combinations (e.g., the tennis problem), and the counting principle (i.e., if one thing can be done in m ways and a second thing can be done in n ways, then the two things can be done together in $m \cdot n$ ways—e.g., the problem in the Problem Corner).

When students use this strategy in natural problem-solving situations before the formal development of these ideas, they gain a valuable background for later understanding.

Include "make an organized list" on your problem-solving strategies bulletin board and compliment students when they use it to solve a problem!

P-9f

Tip Board

Developing Problem-solving Skills

Students develop skill in **problem solving** by acquiring abilities to *understand the problem and question*, to *deal with data*, to *plan what to do*, to *find the answer*, and to *check* to see if the answer makes sense. To help children develop skill in dealing with data, provide problems with these features:

- **Insufficient data,** where students must use sources of data described in the November "Problem Solving" column to find the data needed.
- **Extra data,** where students must select only the data needed to solve the problem.
- **Data alternatives,** where students must make a decision about which of two pieces of applicable data they will use (e.g., which price for the type of gas used when figuring the cost of a trip).

Take a Look

The November 1977 and the February 1982 focus issues of the *Arithmetic Teacher* are devoted to problem solving. They contain lots of good ideas for teachers!

What Works for You?

Do you have some tips for teachers on problem solving? We would like to hear about what has worked in your classroom. If we share our practical ideas, everyone benefits! This part of the Tip Board is reserved for the following:

1. Techniques that you've found useful in teaching problem solving in your class
2. Your favorite problem (a problem that your students have solved and that was one of their favorites)

Give It a Try!

Help students build problem-solving skills by making new problems from a given problem. Think about the characteristics of the given problem. Create new problems by varying one characteristic while keeping the others the same.

1. Change the problem theme.
2. Change the numbers in the problem.
3. Change one or more of the conditions of the problem.
4. Interchange the "given" and the "to find."
5. Change a combination of the items listed above.

This activity not only helps students learn to focus on the important parts of a problem but is a source of additional problems. For further information, check these references:

Charles, Randall I., and Frank K. Lester, Jr. *Teaching Problem Solving: What, Why and How*. Palo Alto, Calif.: Dale Seymour Publications, 1982. (See the section on "Extending Problems.")

Brown, Stephen I., and Marion Walter. *The Art of Problem Posing*. Philadelphia, Pa.: Franklin Institute Press, 1983.

Classroom Climate

To help motivate students and develop their interest in solving problems, try these techniques:

- **Personalize problems.** Use the names of your students and of local places to make problems come alive!
- **Use students' problems.** Encourage students to choose things they are interested in and write problems about them for other students to solve.
- **Use interesting data.** Use data from the *Guinness Book of World Records*, the newspaper, a catalog, a poll, and so on.

Problem Corner

Students in the early grades might enjoy problems of this type. It helps to make an organized list.

How many different outfits can you make if you have a yellow, a red, and a brown shirt and blue shorts and white shorts?

Part of the Tip Board is reserved for techniques that you've found useful in teaching problem solving in your class. Send your ideas to the editor.

P-99

Problem Solving Tips for Teachers

AN AGENDA FOR ACTION

1985

By **Phares G. O'Daffer**
Illinois State University, Normal, IL 61761

Strategy Spotlight

Find a Pattern

Find a pattern is a useful strategy for solving certain types of problems or for dealing with a part of a problem. (See the use of this strategy in the "koala bear" problem in the October 1984 "Tips.") Consider the following problem to see how this strategy might be used.

Problem: A rich neighbor (obviously interested in mathematics!) gave Samantha a choice of $600 for a 16-day house-painting job or 1 cent the 1st day, twice as much the 2d day, and so on, doubling the amount each day. Which arrangement should Samantha choose?

It is natural for children first to record the information in a table (table 1) and extend it according to the "double each day" pattern. Often they find a solution to the problem by extending the table to the 16th day and adding up the pay column. As children solve this problem, or as you discuss solutions, you might ask, "Can you make a 'total pay' column and find a pattern that would save work?"

Table 1

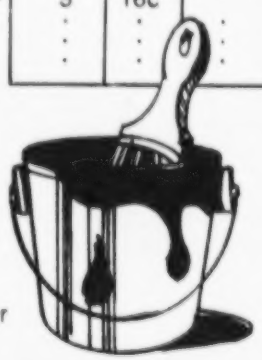
Day	Pay
1	1c
2	2c
3	4c
4	8c
5	16c
.	.
.	.
.	.

When following up on this question, children often discover the pattern shown in table 2 and conclude that the *total* pay for 16 days is 1 cent less than the *pay* for the 17th day!

Table 2

Day	Pay	Total
1	1c	1c
2	2c	3c
3	4c	7c
4	8c	15c
5	16c	
.	.	.
.	.	.
.	.	.

Once given the suggestion to write the pay as a product of 2s, many children find another pattern that makes their work even easier. One child said, "To find the pay, I use 2 as a factor 1 less time than the number of days." Using this pattern, the pay for the 17th day is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 65\,536$c, or $655.36.



Using the pattern in table 2, we find that the total pay for 16 days is $655.35. Samantha should choose the "doubling cents" plan!

Table 3

Day	Pay	Factors
1	1c	1
2	2c	2
3	4c	2·2
4	8c	2·2·2
5	16c	2·2·2·2
.	.	.
.	.	.
.	.	.

Looking for patterns helps children develop their inductive reasoning and problem-solving skills. Include "find a pattern" on your strategies bulletin board and praise children when they use it to find a solution to a problem.

P-9a

Tip Board

Classroom Climate

To improve your students' confidence and ability in solving problems, develop ***your questioning*** and ***listening*** skills.

- Ask questions to help students understand the problem better.
- Ask questions that give them hints for solving when needed.
- Ask questions to give them an opportunity to tell how they feel about problem solving.
- Ask questions that encourage them to describe and analyze their solutions.

Learn your students' needs by listening!

Problem Corner

Students in the middle school might like this one! It helps to look for a pattern.

Benny is starting a computer club. He is the only member now, but his plans are to have each member find 2 new members each month. If his plan works, how many members will the club have at the end of 12 months?

Take a Look

To find ideas from research to help you make decisions in the classroom, see "Research Report: Problem Solving" in the May 1984 *Arithmetic Teacher* and "Update on Research on Problem Solving: Implications for Classroom Teaching" in the February 1982 *Arithmetic Teacher*.

Developing Problem-solving Skills

Students develop skill in problem solving by acquiring abilities to *understand the problem and question*, to *deal with data*, to *plan what to do*, to *find the answer*, and to *check* to see if the answer makes sense. To help children develop skill in determining the reasonableness of an answer, make ***mental math*** and ***estimation*** important parts of your mathematics program. Build the checking skill by assigning exercises like the following:

> Without calculating the exact answer, choose the best estimate. A 4-line ad that runs for 5 days in a newspaper might cost $1.89 per line per day. What is the total cost? (*a*) less than $50; (*b*) more than $60; (*c*) between $50 and $60

Background

Children need to solve a variety of problems requiring appropriate levels of mathematical thinking. Here is a description of some different types of problems that children solve.

One-step: problems solved using a single operation (+ , − , × , ÷)

Multiple-step: problems solved using two or more operations

Process: problems that usually cannot be solved simply by using one or more operations; often require the use of other strategies

Puzzle: problems of the process type that are often solved by a flash of insight or a lucky guess; often purely geometrical or numerical

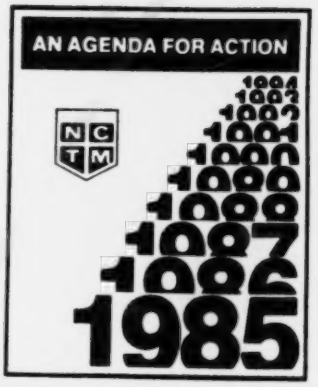
Applied: problems of a real-world type that require more organization, data collection, and the formulation of subproblems; often involve a practical decision

What other categories would you suggest?

Part of the Tip Board is reserved for techniques that you've found useful in teaching problem solving in your class. Send your ideas to the editor.

P-9i

Problem Solving Tips for Teachers



By **Phares G. O'Daffer**
Illinois State University, Normal, IL 61761

Strategy Spotlight

Use Logical Reasoning

Use logical reasoning is a more general strategy than such strategies as make a table, guess and check, or make an organized list. The following problem gives one type of situation where this strategy might be used.

Problem: Ann, Ben, Carlos, and Dina like different types of books: humor, mystery, sports, and adventure. One of Ann's classmates in the group likes mystery books best. Carlos and Dina do not like adventure books. Ben's favorite type of book is sports. Dina did like humor books but has changed her favorite. What is Dina's favorite book?

- First, use a chart to organize the data given in the problem.

	Ann	Ben	Carlos	Dina
Humor				no
Mystery	no			
Sports		yes		
Adventure			no	no

- Then use the information given and logical reasoning to extend the chart.

For example, since Ben's favorite type of book is sports, we can write "no" in Ben's column for every other type of book. After doing this step, we see that Ann is the only possible person to have adventure books as her favorite.

	Ann	Ben	Carlos	Dina
Humor	**no**	**no**	**yes**	no
Mystery	no	**no**		
Sports	**no**	yes		
Adventure	**yes**	**no**	no	no

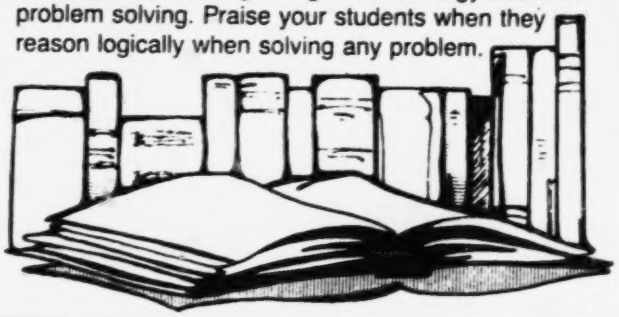
- Use the chart to solve the problem. (Since mystery books are the only type not yet chosen, they are Dina's favorite.)

Some forms of logical reasoning that might be used in solving this problem are these:

1. Since A, then B
2. A *or* B and not A, so B
3. Not both A and B, and A, so not B

Can you find where each form of reasoning was used in the example?

Use logical reasoning is a general strategy useful in problem solving. Praise your students when they reason logically when solving any problem.



p-9j

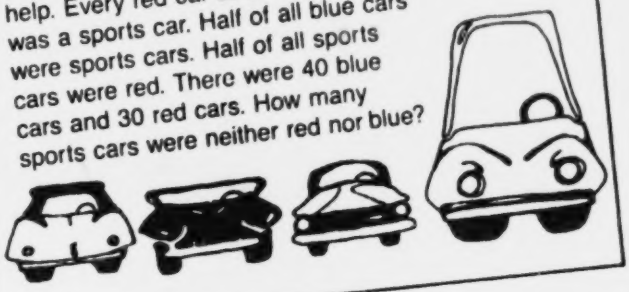
Tip Board

Classroom Climate

- Nothing succeeds like success! Rather than trying to "stump" students with each problem, include many problems that allow students to be comfortably successful "seasoned" with more challenging problems.
- Look for everyday situations about which you can pose questions that stimulate thinking. For example: "At about what time will the hands on a clock be at right angles to each other? How many times do you think this happens while you are at school?"

Problem Corner

Try this with students in grades 6–8. A diagram and logical reasoning will help. Every red car at an auto show was a sports car. Half of all blue cars were sports cars. Half of all sports cars were red. There were 40 blue cars and 30 red cars. How many sports cars were neither red nor blue?



Developing Problem-solving Skills

Students develop skill in **problem solving** by acquiring abilities to *understand the problem and question, deal with data, plan what to do, find the answer,* and *check* to see if the answer makes sense. To help students develop skill in planning, develop the habit of **following up** after a correct solution to a problem has been found.

- Ask students what **strategies** they used.
- Have them explain their **plan and solution**.
- Look for and discuss **alternate solutions** to the same problem.
- Point out and praise instances of their effective use of **thinking skills** in finding the solution.

Take a Look

The booklet *Mathematical Thinking*, a part of the NCTM's Experiences in Mathematical Discovery series, contains some exercises designed to develop thinking skills useful in problem solving. (NCTM, 1977; #174E1, $2.25. Use the order form at the back of the journal.)

Give It a Try!

Try a strategy game to help students develop thinking skills helpful in problem solving. Here is one that has been effectively used with children in the third grade.

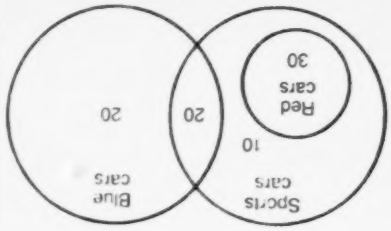
> Two people play this game. Start with 11 counters. On each turn a person picks up 1, 2, or 3 counters. To win, try to make your opponent pick up the last counter.

For additional ideas and other strategy games, see the following sources:

1. Krulik, Stephen, and Jesse Rudnick. *A Sourcebook for Teaching Problem Solving*. Newton, Mass.: Allyn & Bacon, 1984, pp. 55–66.
2. Gallagher, Kevin. "Problem Solving through Recreational Mathematics." In *Problem Solving in School Mathematics*, 1980 Yearbook of the NCTM, pp. 169–77. ■

Answer to Problem Corner

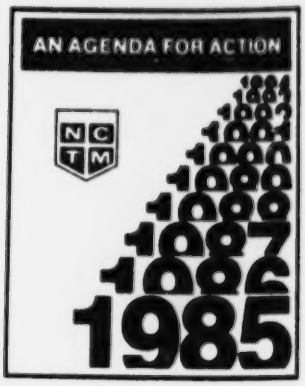
These diagrams fit the conditions of the problem: Ten sports cars are neither red nor blue.



Part of the Tip Board is reserved for techniques that you've found useful in teaching problem solving in your class. Send your ideas to the editor of the section.

P-94

Problem Solving Tips for Teachers



By Phares G. O'Daffer, *Illinois State University, Normal, IL 61761*

Strategy Spotlight

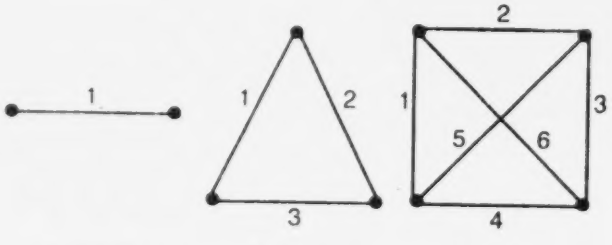
Solve a Simpler Problem

Solve a simpler problem is a strategy that is often used along with strategies such as *make a table* or *look for a pattern* to solve a problem. Consider the following problem to see how this strategy can be used.

Problem: Eric and 6 friends spent a day at the amusement park. At the end of the day, they decided to pair up for roller-coaster rides so that each friend would ride with each of the other friends exactly once. How many rides must be taken?

A useful approach is to pose and *solve simpler problems* by asking, "What if Eric had gone with only 1 friend? 2 friends? 3 friends?"

The following pictures can help us solve these simpler problems.



2 people, 1 ride

3 people, 3 rides

4 people, 6 rides

These three simpler problems help us find a solution to the problem. We make a table and see that the number of rides increases by one more each time. By extending this pattern, we find that Eric and his friends must take 36 rides!

Persons	1	2	3	4	5	6	7	8	9
Rides	0	1	3	6	10	15	21	28	36
		1	2	3	4	5	6	7	8

An older student, when solving this problem, used the three pictures and reasoned, "Suppose we start with n dots (persons). From each dot we can draw lines to $n - 1$ other dots (since we can't draw a line from a dot to itself!). This is $n(n - 1)$ lines (rides). But this approach would be counting 2 lines for every pair of dots (from A to B and from B to A), so we must divide by 2. The formula for n persons is $n(n - 1)/2$ rides!"

Other ways to use the *solve a simpler problem* strategy include these:

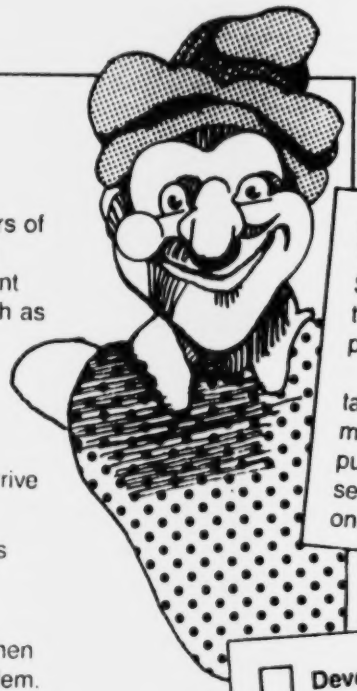
- Change the numbers in the problem (e.g., from decimals to whole numbers) so that the conditions of the problem are more obvious. Solve this simpler problem. Then use the same solution process to solve the original problem.
- Formulate and solve simpler subproblems and use their solutions to help solve the original problem.

Tip Board

Classroom Climate

To alleviate the anxiety many children have about solving problems, use puppets or posters of cartoon characters to help them develop a more relaxed, confident approach. Emphasize ideas such as these:

- Problem solving takes time and patience.
- It is OK to make mistakes. Sometimes errors help you arrive at a solution.
- A person who makes progress toward a solution has been successful.
- It's really fun and satisfying when you try hard and solve a problem.



Solution to **Problem Corner**, see page 52

Problem Corner

Students in grades 3–5 might like this one. It helps to solve a simpler problem.

Tina wanted to seat guests at tables at an outdoor barbecue. How many people can she seat by pushing together 27 card tables that seat 1 person on each side to make one long table?

Developing Problem-solving Skills

Students develop skills in ***problem solving*** by acquiring abilities to *understand the problem and question*, to *deal with data*, to *plan what to do*, to *find the answer*, and to *check* to see if the answer makes sense. To help children develop skill in understanding the problem, try the following:

- Give a story or a problem and have children act out the situation and phrase the question in their own words. Use objects when helpful.
- Give middle-grade children a number sentence and encourage them to formulate a problem that can be solved using the sentence. Post children's problems on a bulletin board for other children to solve, or make a weekly newsletter containing problems that students have created for others to solve.

Give It a Try!

A "priceless" open-ended problem like the following can help some children change their mind-set about problem solving and decide that (a) they *can* solve problems and (b) it is fun to do!

Match amounts of money with letters of the alphabet as shown:

a	b	c	d	e	f	g	h	i	j	k	l	m
1¢	2¢	3¢	4¢	5¢	6¢	7¢	8¢	9¢	10¢	11¢	12¢	13¢

n	o	p	q	r	s	t	u	v	w	x	y	z
14¢	15¢	16¢	17¢	18¢	19¢	20¢	21¢	22¢	23¢	24¢	25¢	26¢

How many $1 words can you find? (Is *turkey* a $1 word?)

For additional discussion on changing children's mind-sets and some activities that can be used for this purpose, see the following sources:

Meyer, Carol, and Tom Sallee. *Make It Simpler—a Practical Guide to Problem Solving in Mathematics*, pp. 10–16. Reading, Mass.: Addison-Wesley Publishing Co., 1983.

Souviney, Randall J. *Solving Problems Kids Care About*, pp. 18–21, 39–43. Santa Monica, Calif.: Goodyear Publishing Co., 1981.

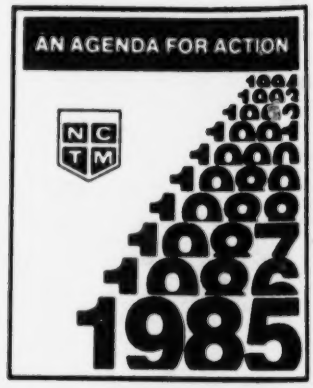
Take a Look

The thirty-seventh NCTM Yearbook (1975), *Mathematics Learning in Early Childhood*, has a section on problem solving for young children (pp. 73–90) that contains some excellent ideas and problems for children aged three to eight.

Part of the Tip Board is reserved for techniques that you've found useful in teaching problem solving in your class. Send your ideas to the editor of the section.

P-9m

Problem Solving Tips for Teachers



By **Phares G. O'Daffer,** *Illinois State University, Normal, IL 61761*

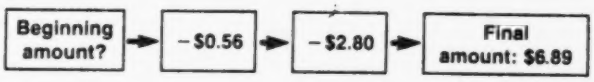
Strategy Spotlight

Work Backward

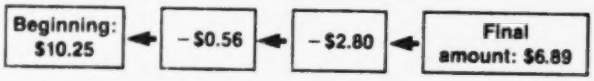
An example of the ***work-backward*** strategy was given in conjunction with the draw-a-picture strategy in the October 1984 Tips section. Consider the following problem to see another situation in which this strategy might be used:

Problem: Sue counted the amount of money she had earned during the week and wrote it on a slip of paper. Then she bought some postcards for $0.56 and a pen that cost $2.80. After her shopping she had $6.89 left, but she had lost the slip of paper. What was the amount written on the paper?

This problem can be viewed as follows:



To find the beginning amount, we use the idea that addition and subtraction are inverse operations and think about reversing the "flowchart" above.



Using the *work-backward* strategy, we begin with the final amount and reverse the operations to arrive at the first amount. It may help some children actually to make a "flowchart" like the one shown and reverse it to solve the problem.

Problems that can be solved using a work-backward strategy can often also be solved by writing and solving an equation. For the previous problem we could write the following:

$$\begin{aligned} A - 0.56 - 2.80 &= 6.89 \\ A - 3.36 &= 6.89 \\ A &= 10.25 \end{aligned}$$

Early experience in solving problems using a work-backward strategy can help children begin to develop an understanding of the process of translating verbal descriptions into equations, as well as of the technique of using inverse operations to solve equations.

Include the *work-backward* strategy on your classroom ***Problem-solving Strategies*** poster or bulletin board, encourage students to use it when appropriate, and compliment them when they use it effectively in a solution to a problem.

Tip Board

P-9n

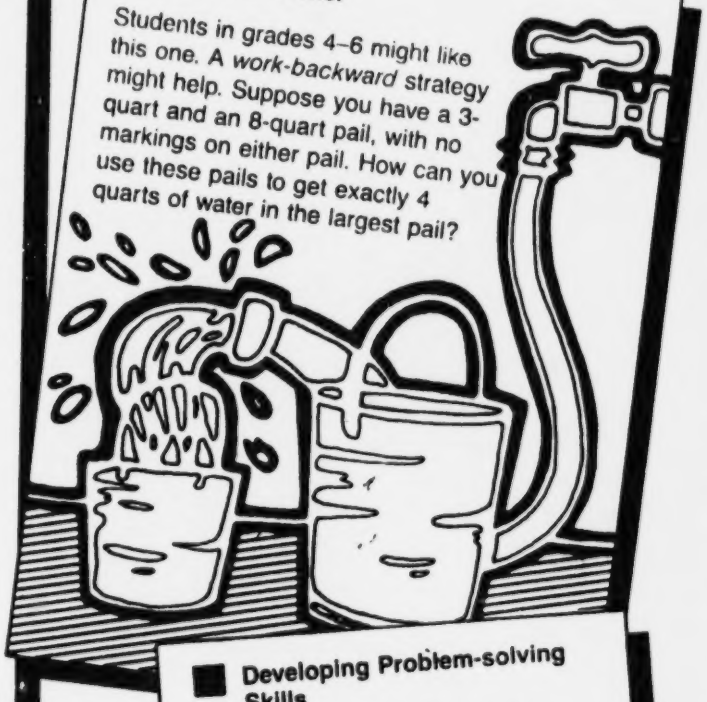
Classroom Climate

Find ways to help children experience problem-solving situations that are *realistic:*

- Groups of adults often *work together* in solving problems and *communicate* with each other about their work. Let students do this!
- Adults regularly *use calculators* to aid in solving problems. Find similar opportunities for students!
- Adults often have to *search for data* to use in solving problems. Give students problems that require realistic and interesting data-search experiences!
- Adults are sometimes asked to *write a description of their solution* to a problem, so that others can understand their thinking. Ask children to record their work and write about their solutions!
- Adults sometimes make progress toward a solution to a problem by *solving only part of the problem.* Others work on other parts, and their solutions are combined to arrive at the final solution. Can we arrange experiences like these for children?

Problem Corner

Students in grades 4–6 might like this one. A *work-backward* strategy might help. Suppose you have a 3-quart and an 8-quart pail, with no markings on either pail. How can you use these pails to get exactly 4 quarts of water in the largest pail?



Developing Problem-solving Skills

Students develop skill in **problem solving** by acquiring abilities to *understand the problem and question,* to *deal with data,* to *plan what to do,* to *find the answer,* and to *check* to see if the answer makes sense. To help children develop skill in finding the answer, help them develop these **mental-math** skills and use them when solving selected problems.

- **Look for numbers that go together.**
  (For example, look for a pair of numbers that add to 100 in the sum 47 + 78 + 53.)
- **Change a number; adjust someplace else.**
  (For example, *add 2* to change 52 − 24 to 54 − 24, or 30; adjust by *subtracting 2* to get 28.)
- **Break a number into parts.**
  (For example, find 34 + 28 by adding 4 + 8 = 12 to 30 + 20 − 50. The answer is 50 + 12, or 62.)

Take a Look

Find out how to make a **Problem-solving Bulletin Board** that encourages students' involvement. See pages 127–35 in the NCTM's 1980 Yearbook, *Problem Solving in School Mathematics.*

Solution to the "Problem Corner":

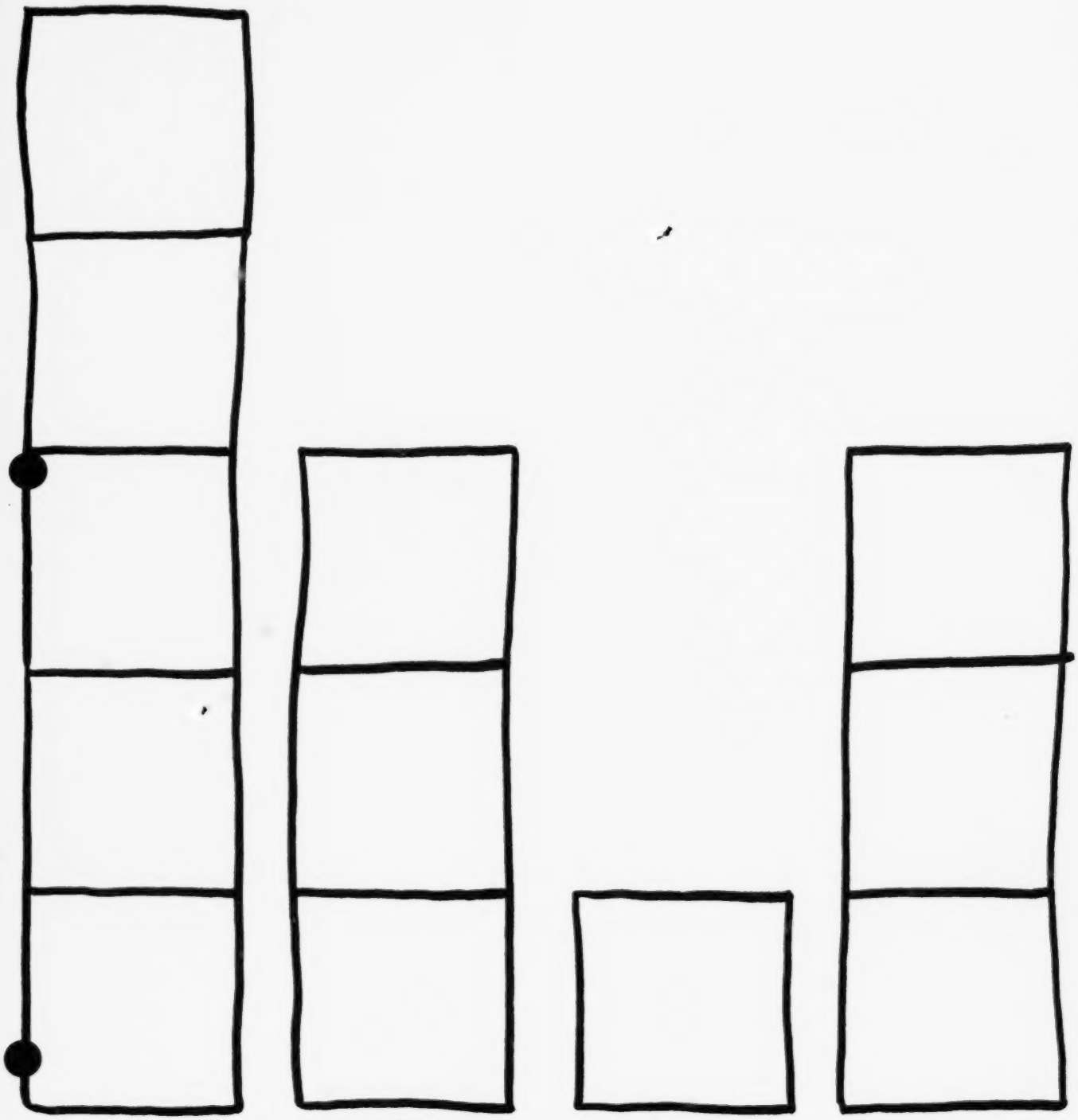
We start by thinking about 4 qt. in the 8-qt. pail. We can get 4 qt. by getting 1 qt. and 3 qt. But how can we get 1 qt.? Aha! By filling the 3-qt. pail 3 times and pouring it into the 8-qt. pail, we are left with 1 extra qt. in the 3-qt. pail. Now we empty the 8-qt. pail and pour in the 1 qt. Then we fill the 3-qt. pail and pour that in to arrive at the desired 4 qt.

Part of the Tip Board is reserved for techniques that you've found useful in teaching problem solving in your class. Send your ideas to the editor of the section.

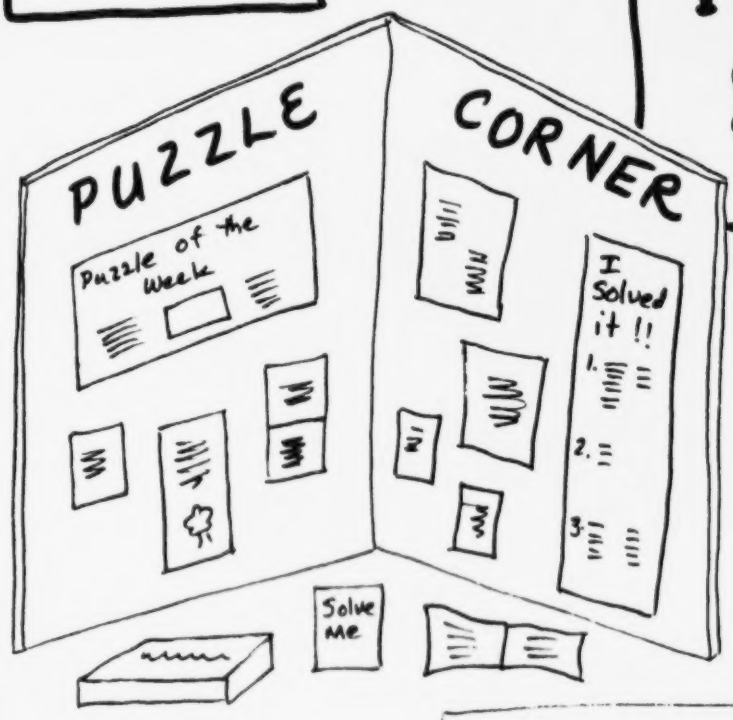
OTHER APPROACHES to PROBLEM SOLVING

- Allow time and opportunities.
- Encourage children to make and solve their own problems.
- Provide a place and motivation for individual problem solving.
- At times, practice on a single step in the problem-solving process.
- Gather and share materials for problem solving.

OUR FAVORITE ANIMALS

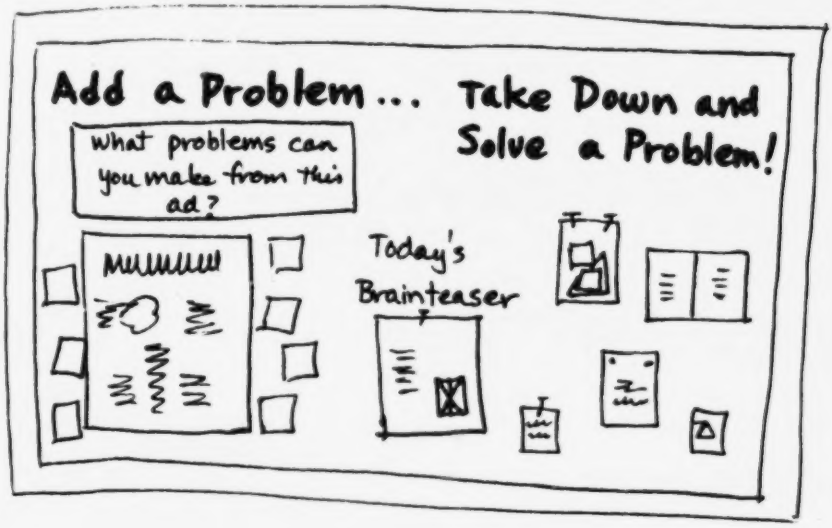


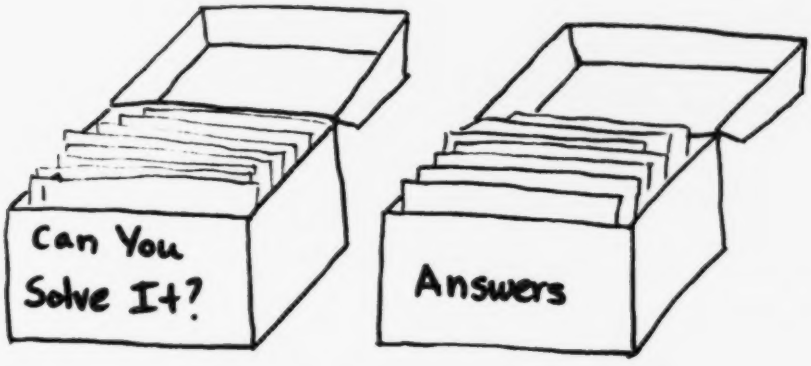
Places for PROBLEM SOLVING



This self-standing center could set on a table top or on the floor. Multipiece puzzles store in boxes or plastic bags.

A bulletin board invites students to contribute and solve problems.





Add items easily to problem and answer files.

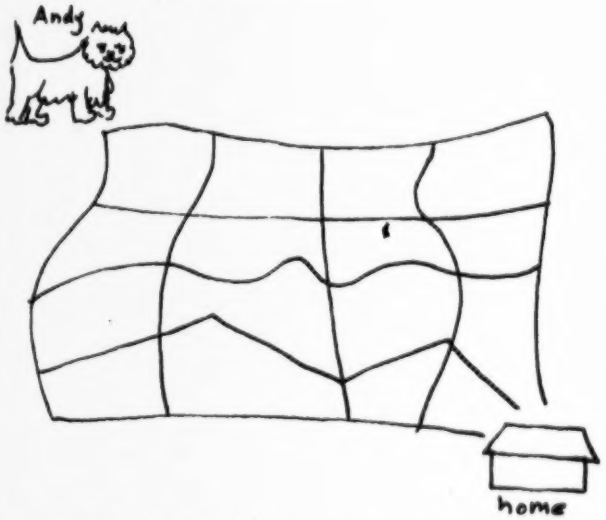
Sources and Resources for PROBLEM SOLVING

- Books, magazines, newspapers, catalogs
- Writing own seasonal, current event, or local interest problems
- Professional meetings
- Other curricular areas
- Oral presented problems

more PROBLEMS

1. A carpenter makes only 3-legged stools and 4-legged tables. At the end of one day, he had used 31 legs. How many stools and how many tables did he make?

2. Find at least 10 different paths for Andy to use to go home.



Which paths are shortest?

3. How many different ways can you find to add <u>5 odd</u> numbers to get a <u>sum of 20</u>? You can use a number more than once.

4. You need 17 lb. of fertilizer. What can you buy to get at least that amount for the lowest cost?

5. Vicki needs $20.00. Here are some jobs she can get:

wash dishes for the family -- 50¢
clean kitchen -- $2.00
baby sitting -- $1.50 per hour
walk neighbor's dog -- 75¢ a trip
mow lawn -- $4.00

What are some ways she can each the $20.00?

6. Divide the square into 2 equal parts. Can you find at least 5 ways to do it?

EVALUATION OF YOUNG CHILDREN'S PROGRESS IN MATHEMATICS

In this final workshop session, participants will receive information on the importance and nature of evaluation. They will explore these related aspects of teaching and give examples of using each aspect. They will study means of evaluating and recording children's progress in mathematics. The workshop leader should:

1. Present information on the nature and importance of evaluation.
2. Discuss physical, verbal, and symbolic aspects of teaching and invite participants to offer examples of these aspects.
3. Present and discuss ways to have children demonstrate their knowledge of concepts and skills.
4. Share ways to focus on children's problem-solving abilities.
5. Discuss assessment of children's attitude toward school mathematics.
6. Ask participants to share other ways of evaluating children's math achievement or attitudes toward mathematics.
7. Prepare materials in advance:

   Transparencies E-1 through E-4, E-6 through E-10.

   Handouts for participants E-5 and E-11.

   Secure any math evaluation guides or checklists already in use in the district.

1. Why is evaluation important? Evaluation is an important aspect of any instructional program. (Transparency E-1.) Evaluation lets educators assess progress toward objectives and goals. When it is aimed toward different aspects of the instructional program, evaluation gives teachers and assistants insight into their classroom behaviors and practices. Evaluation can also "get at" attitudes of learners. Evaluation procedures are important to learners, too; even young children, as they demonstrate mastery of skills and concepts, need to see their own growth.

   Evaluation can be quantitative or qualitative. (Transparency E-2.) Quantitative observations and measures give educators numerical data on aspects of instruction. For example, teachers might find that a child adds correctly in 85 percent of a set of problems involving two-digit numbers regrouping. A child might say the names of 4 out of 5 geometric figures correctly. Qualitative data can also be gathered on classroom procedures, attitudes, and processes. Teachers and assistants might use checklist items to determine pupils' attitudes toward math for example. If they find that a pupil almost always chooses math activities in free time, volunteers frequently to demonstrate how to do problems, and "stays with" word problems and puzzles until they are completed, adults might assume that the child shows positive attitudes toward mathematics.

2. Physical, verbal, and symbolic aspects of mathematics. (Transparency E-3.) Physical, verbal, and symbolic aspects of mathematics are closely related. All three aspects should be constantly used. The physical aspect lets us represent mathematical relationships with counters, pictures, and other materials. The physical aspect also concerns the handling of materials to show changes or the use of measuring tools. For example, to represent subtraction, we might show 8 pennies, take 3 away, and count the 5 that are left. We might take a paper rectangle and cut it on a diagonal to show that two congruent triangles result. The verbal aspect occurs when we "talk mathematics." Both children and adults need to verbalize about math operations, relationships, and problems. As we record our work in mathematics, we use symbols. Numerals and operation signs are the most commonly used symbols for work with young children.

   The three aspects — physical, verbal, and symbolic — complement each other and add meaning to each other. Without discussion and recording with symbols, physical manipulations of objects don't mean much. Talk about mathematics alone is ineffective; we must add the use of physical representations and written symbols or pictures. Symbols, shared world-wide, are extensions of physical objects and words. All three aspects should be used for effective teaching.

Let's illustrate the three aspects (Transparency E-4). As she introduced the fractions fifths and sixths to a second grade class, Mrs. White used squared paper. She and the children colored in various patterns, named some fractions, and had the children name the fractions that were shown. She showed the children how to record fractions and let volunteer children write fractional names on the chalkboard as they were used. Next she used some colored balls in a bag. The children took turns pulling out five and six balls at a time. They made verbal sentences about the balls: "Two out of 5 are blue; 1/5 are green; 2/5 are red" and recorded data on a chart using fractions.

	Fractions of			
Whose turn?	Blue	Red	Yellow	Green
Jay	1/5	1/5	2/5	1/5
Samantha	2/6	1/6	2/6	0/6
Kaye	2/5	0/5	3/5	0/5

Of course the paper squares and balls were physical representatives of number relationships we call fractions. Discussion about the physical objects provided the verbal aspect of the activity. Both the children and the teacher did lots of talking. The symbolic aspect was used as the teacher and children recorded numerals to represent the fractions. What are some other examples of using the three aspects?

In thinking about instructional techniques, teachers and assistants might use the questions on Handout E-5 to assess their use of physical, verbal, and symbolic aspects. Participants could discuss various questions, giving examples of recent uses of the aspects in their classrooms. Invite participants to add items to the handout.

3. Children demonstrating concepts and skills? "Show me." and other useful phrases. It's easy to think of mathematics as a yes-no/right-wrong subject. But to assess children's understanding and thinking about mathematics, we must probe beyond simple answers.

Adults can often open conversation with the phrase,"Show me." If a pupil says that 4 + 2 = 7 and is asked to "show me," with felt board counters, she almost always finds her own error and says, "Oh, OK, it's 6." When kindergarteners are asked "Which set is bigger?," they should show adults how they know, often by matching objects from one set with those from another set until some (from the larger set) remain without partners. Adults can see if children understand the regrouping process in subtraction by asking, "Show me how you regrouped." Often children will show and tell about the process as well. Children should be asked to show work that's correct as well as incorrect. Adults should watch answers to "show me" questions with an open mind. Sometimes children show solutions to problems in ways that are correct but different than the ways adults expected.

Other phrases that usually generate discussion or action are these:

- "Can you do that another way? How?"
- "What patterns do you see?"
- "What does this answer mean/tell us?"
- "How could you explain/show that to somebody younger?" (Second and third graders often like to tackle this question.)

What other such questions have you found effective in evaluating children's understanding?

4. Informal evaluation of student progress. Teachers use informal means of evaluating children's progress and difficulties constantly. They observe children's work. They listen and judge answers as children participate in class discussions and activities. Adults judge children's work habits, too.

Informal records may be kept by writing anecdotal notes about specific incidents and keeping these in the children's folders. If these notes are dated, they can be looked over later to see patterns of behavior and achievement.

Adults may also want to keep checklists for the entire class. They might specify certain expected behaviors and check these off as children evidence the behaviors. For instance, in working with first graders on numerals 10-19, teachers or assistants might use a form such as the one shown on Transparency E-6. Since all names are kept together, the form is easy to mark. It shows individual as well as group and class data. The teacher or assistant might mark such a form as children work in groups with learning activities and as they do classwork. Use of such a form shows individual mastery and weaknesses far more specifically than does a more global score on a unit test.

(Teachers and assistants might now want to discuss record keeping in the school district. Show and discuss district record keeping and checklist forms. Development of checklists for any topic is not difficult and can result in easy-to-use but informative forms.)

5. Evaluation of children's problem-solving abilities. The Agenda of The National Council of Teachers of Mathematics addresses the difficulty of evaluation of children's problem-solving abilities (Transparency E-7):

> "An emphasis on problem solving demands more flexibility and creativity than is possible within the restriction of most current test formats."

Teachers and assistants need to go beyond the pencil-and-paper formats of more tests and gather observational data. Educators must remember that problem solving is a developmental process that requires much time. Measurement of the process should be continuous — over the entire school mathematics program.

Evaluation of problem solving should be based on the goals of individual teachers as well as goals of the school system. If the program, based on goals, includes both routine and nonroutine problems, then measures of evaluation must include both kinds of problems. If the program involves the use of pictures, charts, and graphs in problem-solving, then assessments must give children opportunities to display their skills in these areas.

To measure children's problem solving abilities, adults might also (Transparency E-8):

Present problem-solving situations and see how children meet them. Take time to let small groups of students help with problem situations that arise in the classroom and evaluate their skills as they work. Ask children to help occasionally with lunch money, counting attendance, or ordering supplies. Let them help you work with arranging and counting equipment for science experiences. As children work with problems in small groups, circulate and consider points such as these (Transparency E-9):

- Is there evidence that children read problems carefully?
- How do children begin to attack problems?
- Do they try another strategy if their first one fails? Do they only try the skill most recently taught?
- Are children consistent and persistent in applying a strategy for problem-solving?
- What strategies does each child use most?
- If careless errors are made, when and why are they made?
- Do children stay with a problem until it is finished?
- Do they concentrate on their task?

- How quickly do they ask for help?
- Do children use manipulative materials?

Interview children. (Back to Transparency E-8) Talk to children about the process of problem solving. Present a child with a problem and have him or her discuss the thought patterns involved. Ask children why they made the decisions they did in problem solving. Children may also be interviewed in groups as they work together. If you tape-record responses, you can play back the conversation and analyze student thinking carefully. Other groups may also profit from listening to the audiotapes.

Use a checklist of different problem-solving strategies. As children use strategies such as making graphs, drawing pictures, or solving simpler problems, check off the use of these strategies.

Pencil and paper tests. Ask children to work independently on both routine and nonroutine problems. As you check their work, note not only the correctness of answers, but also the approaches that were used. Even for young children you might give credit for correct methods as well as correct final answers.

Keep samples of children's work. Samples of work can be filed — by children as well as by adults. These samples are useful in explaining the quality of children's work in parent conferences. They are also useful in talking to the children about their progress, strengths, and weaknesses.

6. Assessing attitudes. Children's attitudes are very important. Children who enjoy math and are confident about their abilities are a pleasure to work with because they try hard and often achieve better than those who seem to dislike math and are afraid of it. Teachers and assistants can help to foster positive attitudes with their own enthusiasm, by offering constant encouragement, and by varying routines so that mathematics is not boring.

Achievement builds confidence in children; confidence builds future achievement. It's like a spiral of increasing competency and pride (Transparency E-10). By helping children to be successful at mathematics through individual and small group help, teachers and assistants build positive attitudes too. If children can set and reach some of their own goals (discussed previously in material on drill on basic facts), children can see that they are learning and making progress. Letting children help with record keeping demonstrates confidence in children's abilities to keep track of their own progress, and lets children see that they are making headway or assess where they are not doing as well.

Parents are important in developing and reinforcing children's attitudes towards matehmatics. Where possible, tell parents of children's achievements. Stress their children's strong points. If parents express negative feelings about math ("I was never any good at math."), try to develop the idea that their children can and are succeeding. Work with parents to support children's achievement through some practice and use of manipulatives and problem solving at home.

To try to assess children's attitudes toward mathematics, you might use a simple checklist like the one presented in Handout E-11. What items would you add to the list?

7. Other effective ways of evaluating children's progress in math. Ask participants to share other means of evaluation that they have found effective. Discussion of both things to look for and simple efficient means of recording data will be valuable.

EVALUATION

Educators assess progress toward goals & objectives.

Insight into classroom procedures.

Students assess own progress.

Educators examine children's attitudes.

Evaluation can be

Quantitative

- Use numerical data

Qualitative

- Assesses processes, attitudes, class procedures

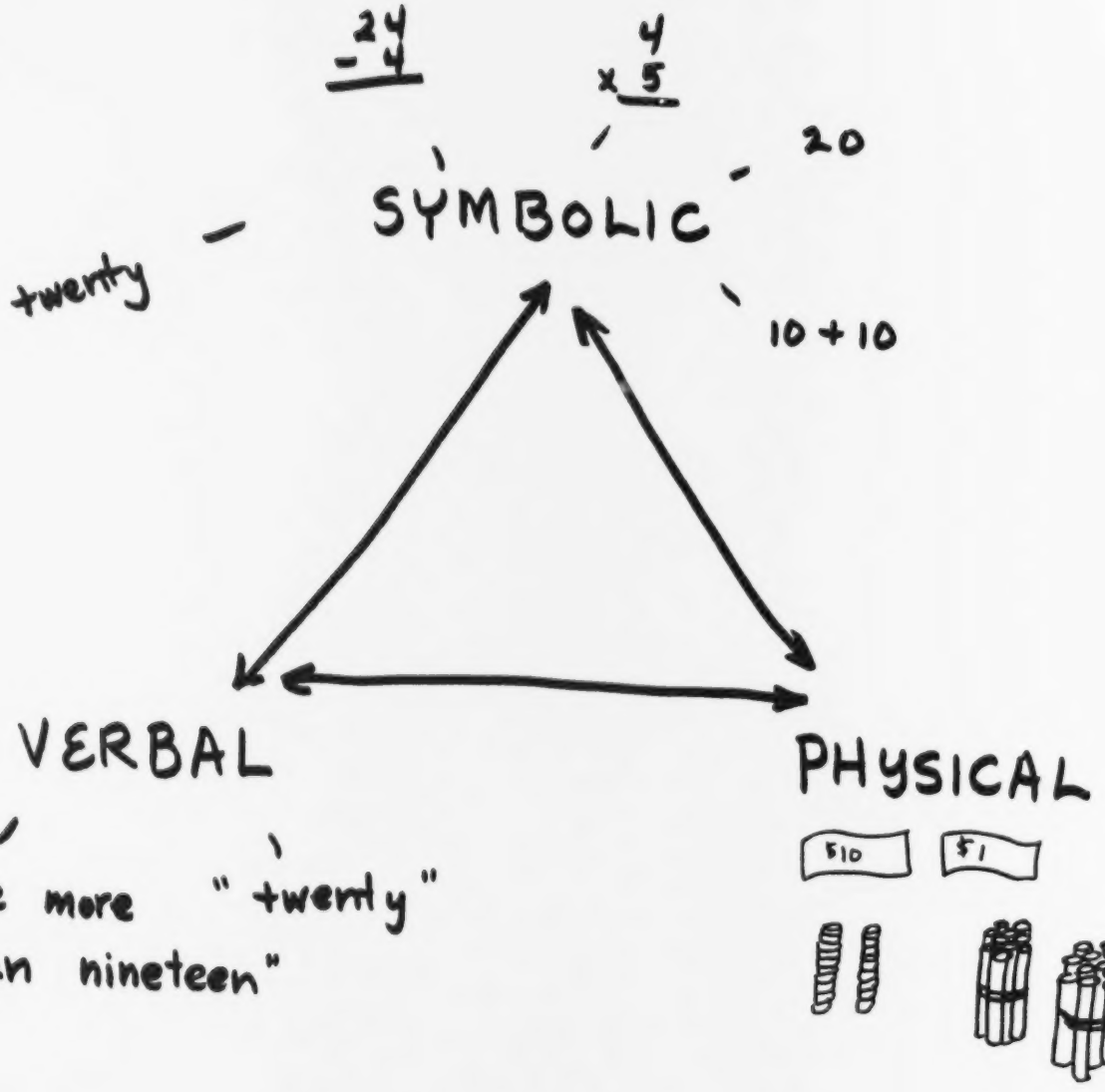
Quantitative Evaluation

- 4 out of 5 right
- 90%
- 7/10 of problems right

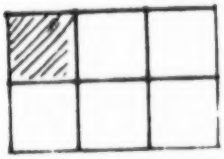
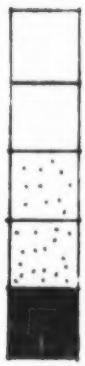
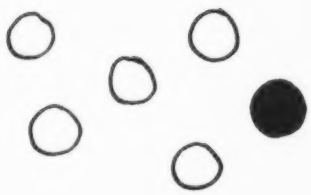
Qualitative Evaluation

- effort
- methods of problem solving
- use of verbalizations
- interest
- drill & practice

3 Interrelated Aspects of Mathematics



Introducing FRACTIONS

PHYSICAL	VERBAL	SYMBOLIC
	"one out of six is striped."	$\frac{1}{6}$ or 1/6
	"$\frac{2}{6}$ is gray."	$\frac{2}{6}$
	"$\frac{3}{6}$ or $\frac{1}{2}$ is white"	$\frac{3}{6} = \frac{1}{2}$
	"$\frac{1}{5}$ is gray and $\frac{1}{5}$ is white."	$\frac{1}{5}$ - gray $\frac{1}{5}$ - white
	"$\frac{2}{5}$ is dotted; $\frac{1}{5}$ is black."	$\frac{2}{5}$ - dotted $\frac{1}{5}$ - black
	"Jay got 2 gray, 3 white, and one black."	
	"$\frac{2}{6}$ are gray, $\frac{1}{6}$ is black, $\frac{3}{6}$ are white."	

gray	black	white	red
$\frac{2}{6}$	$\frac{1}{6}$	$\frac{3}{6}$	$\frac{0}{6}$

EVALUATING USES OF PHYSICAL, VERBAL, AND SYMBOLIC ASPECTS OF CLASSROOM INSTRUCTION

1. How have I used physical, verbal, and symbolic aspects of teaching in my classroom:
    a. yesterday?
    b. last week?
    c. throughout the last month?
2. What was the nature of the physical aspect?
    a. Did I use a variety of representations of mathematical concepts or relationships?
    b. Who handled the manipulatives?
    c. About what proportion of the time period did we use the physical aspect?
    d. How did I accompany physical work with verbalizations and symbols?
3. What was the nature of the verbal aspect?
    a. Who did the talking?
    b. What proportion of the children got to talk?
    c. Did I rely on volunteers for responses or did I ask particular children?
    d. Were children's answers short or did I get them to explain or elaborate on their answers?
    e. Did I use some open-ended questions where several children could participate?
    f. How did verbalizations complement physical and symbolic aspects?
4. What was the nature of the symbolic aspect?
    a. Did both the children and I use symbols? How?
    b. Did we identify, write, read, and talk about symbols? What were other ways to use and explore symbols?
    c. How did I stress or expect neatness and accuracy as the children and I worked with symbols?
    d. How did work with symbols add to the physical and verbal aspects of learning activities?

Sample Checklist
NUMBERS 10 - 19

Behavior \ Child	Group A: Betty	Andy	Carol	Kneeky	Daryl	MaryJo	Joey	Group B: Ken	Alan	Dex	Randy	Joannie	Pearl	Group C: Kelly	Vicki	Billy	Dan	Group D: Casper	Nan
Counts 10-19 objects rationally																			
Arrange objects on numeral cards (8/10 correct) [18] [dots]																			
Matches numeral-number cards (8/10 correct) [13] [dots]																			
Writes numerals legibly																			
Told 2 numbers 10-19, arranges objects with each "fourteen" IIIIIIIIIIIII																			
Chooses larger or smaller of 2 nos. Chooses numeral card for each "seventeen" > "fifteen" [17] [15]																			

- from the Agenda for Action for the 1980's

National Council of Teachers of Mathematics

"An emphasis on problem solving demands more flexibility and creativity than is possible within the restrictions of most current test formats."

Assessing Problem Solving Abilities

- Present problem-solving situations and observe how children work.
- Interviewing
- Use of checklists
- Pencil and paper tests
- Samples of children's work

Observing Children as they Solve Problems

- Careful reading
- Attacking, starting problems
- Consistence
- Persistence
- Strategies
- Finishing work
- Concentration
- Asking for help
- Use of manipulatives

Widening of positive attitudes
and skills
internal motivation
pride
success
external motivation
greater achievement
more work
limited achievement

SOME QUESTIONS ABOUT MATHEMATICS

(ATTITUDE SCALE FOR CHILDREN)

Fill in the mouth on each little face to show how you feel. If you agree with an item, mark . If you don't agree, mark . If you just don't care about the item, mark . (Teachers should read each item aloud to a group of 8 to 10 children and allow time for children to mark the scale. Stress that there are no right or wrong answers; you just want to know how the children feel about each item.

1. Math is important.

2. I'm afraid of problem solving.

3. Math is my favorite subject.

4. I like to plan how to spend money.

5. Work with numbers is boring.

6. I feel good when I work math problems.

7. I would like to use a calculator or computer to work with numbers.

8. My parent(s) like math.

9. If I had a choice, I wouldn't study math.

10. I like to work with clocks and calendars.

11. I don't like to study and draw circles and rectangles and triangles.

12. I don't care if I do well in math or not.

13. I like to check my answers after I do my math work.

14. Counting things is boring.

15. Math makes you think hard.

16. I like to show others how to do math problems.

17. I don't like to show math work on the chalkboard.

18. When I have extra time, I like to do math activities.

(Teachers may add pertinent items.)

MAKE IT, TAKE IT

After a brief introduction to materials and procedures, participants will spend their time in this session making teaching — learning aids that they can take back for use in their classrooms. The workshop leader should:

1. Secure materials for teaching — learning aids to be made during the session (or specify in advance the materials that teachers and teaching assistants should bring).

2. Introduce materials that can be made during the session.

3. Outline procedures for use of materials and cleanup.

4. Support participants' work during the session.

5. Have participants briefly share or display their work at the end of the session.

6. Prepare patterns and handouts prior to the presentation:
   - Participants' handout M-1.
   - Transfer patterns M-2 through M-5. Transfer parts of M-5 to cardboard or poster board for participants to trace around.
   - Materials as listed — if the school district can supply laminating film or clear contact paper for covering most teaching aids, materials will last much longer. Rulers, extra scissors, and glue will be helpful also.
   - Make M-4 into transparencies as well as patterns. Show participants how to project the transparencies onto a poster board or construction paper and trace the image if they wish to make a larger-sized game board than the pattern.
   - Include materials from other workshops in this session if participants seem interested in materials and have not made them previously.

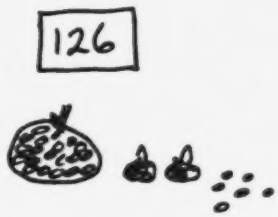
SOME MULTI-PURPOSE MATHEMATICS MATERIALS
FOR USE WITH YOUNG CHILDREN

Counters

- Use for counting, showing place value, representing numerals, representing problems with operations.
- Materials: small objects that can be "bundled" and "unbundled" — for example, toothpicks or popsicle sticks held together with rubber bands, soda bottle tops attached together with pipe cleaners, or beans packaged in plastic wrap held by rubber bands.
- Make several sets of counters — single counters for kindergarten, tens and ones and a few hundreds for 1st grade, hundreds, tens, ones, and a few thousands for 2nd and 3rd grades.
- Pupils can work with counters individually or in small groups.

78 means 7 tens and 8 ones

popsicle sticks bundled with rubber bands



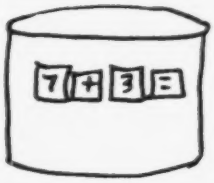
beans in plastic wrap

73 bottle caps --
one set of ten separated to make 13 "singles" or ones.
59 "taken away." 14 left

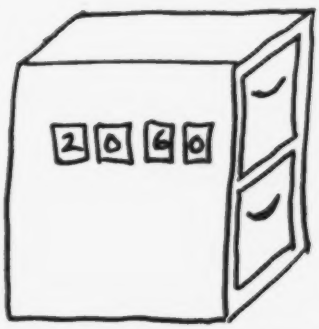
Magnet Numerals

- Use for forming numerals to read, ordering numerals, setting up operations to solve mentally or on paper.
- Materials: small pieces of tagboard or manilla folder, magnetic tape (if not self-sticking, glue or affix), 3 lb. coffee can, spray paint.
- Spray paint the coffe can. Prepare numerals cards. Affix small bits of magnets to the back of each card. Normally a 2 cm x 2 cm piece of magnetic tape will hold the card securely to a steel surface.

Let the magnet "stick to" the can --- or use them on a file cabinet.



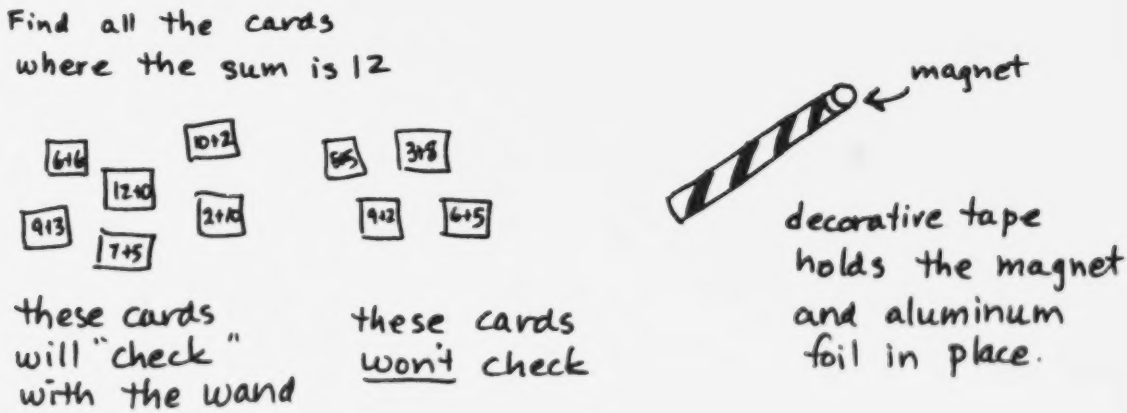
"ten"



"two thousand sixty

The Magic Wand

- Use for any sort of question where answers can be sorted into 2 piles — rectangles or circles? Is the sum equal to 12? Are the coins worth 25¢? Are the numbers multiples of 3?
- Materials: construction paper, paper clips, transparent tape, strong magnet, short dowel or thin cardboard tube (from a hanger), aluminum foil, decorative tape, pictures of coins from old workbooks if desired.
- Make pertinent question cards. Put paper clips on the correct answers but not on the incorrect answers. Make the magic wand by taping the magnet to the dowel or tube, then decorating the tube with foil and tape. If you make more than one set of cards, color-code them so they don't get mixed up.
- Children sort the cards into two stacks. They then check the stacks by touching the magic wand to the correct answers. The wand should pick up the correct cards, but not the others!



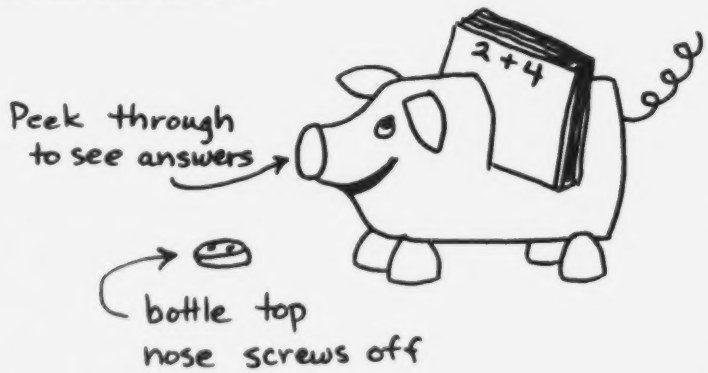
Big Numerals

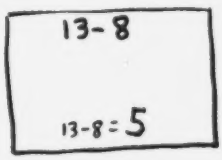
- Use for bulletin boards and displays, rubbing, tracing around, decorating with correct number of pictures.
- Materials: construction paper or posterboard, sandpaper (if making "feely" numerals), glue.
- Trace around the patterns (M-2) and cut out the numerals. If making sandpaper numerals, cut sandpaper with sturdy scissors and glue to posterboard rectangles.

Put numeral below thin paper. Add correct number of shapes. Rub over it with crayon.

Math Pig

- Use for individual practice with basic facts.
- Materials: Well-washed bleach bottle, felt or permanent markers, glue, stryofoam egg carton, brads, manilla folders or light posterboard, knife or sturdy scissors, pipecleaner.
- Decorate the bleach bottle to look like a pig. Fasten egg carton section feet to the bottle with brads. Be sure to decorate the nose. Cut a 1-2 cm slit in the pig's back as shown. Attach a curly pipecleaner tail. Prepare a series of cards to fit in the pig's back. The problems should show above the slit, and the answers should be visible when a child peeks through the pig's nose to check.
- A child inserts a pack of cards into the pig's back. He or she says the answer, then checks by looking through the nose hole. He or she puts the card correctly answered in the back of the pack and tries the next problem. Cards missed should be put aside, studied, then tried again.

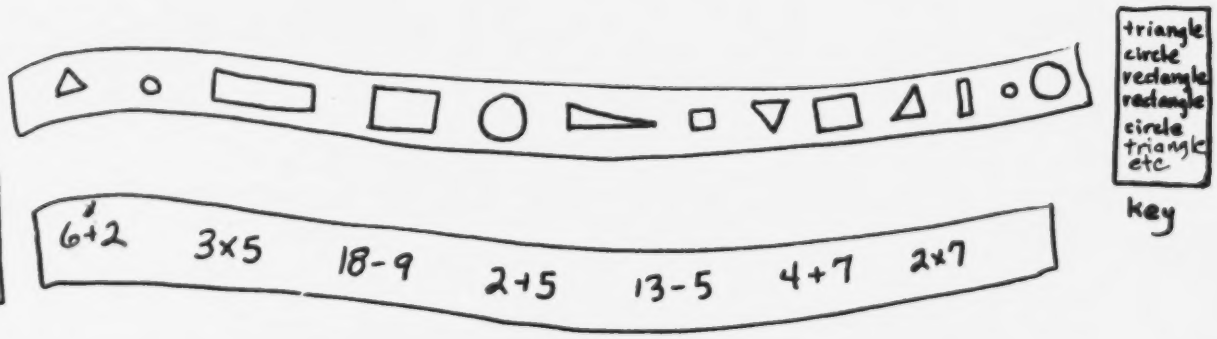




Example of card

Fact Walk I

- Use for practice facts, names of shapes, money values, telling time, and other skills.
- Materials: Shelf paper or sentence strip paper, permanent markers, note cards, perhaps old workbook pictures of coins or clocks.
- Write a series of questions on a long strip of paper. Use two or three colors for the questions. Make a color-coded answer key on a note card.
- Attach the strip to a wall. Have children work in pairs. One walks along the strip and says the answer while the other checks. Then the pair reverses roles for more practice.



Fact Walk II

- Use for practice on basic facts, or saying numerals.
- Materials: Construction paper, markers, note cards.
- Trace and cut out child-sized feet (Pattern M-3). Write appropriate facts or numerals on the feet. Make an answer key.
- Let a child arrange the feet on the floor in the order of the answer key. One child walks along the feet, saying answers as quickly as possible. Another child walks along and checks. Adults may want to "time" children on groups of facts and let the children see their progress as they master the facts.
- Feet may also be made of contact paper and affixed to the floor.

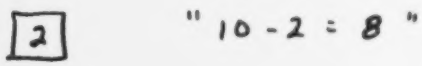
Numeral Cards

- Use for ordering, work on basic facts, matching with counters, making and reading numerals.
- Materials: Construction paper on note cards, markers.
- Write large, clear numerals on cards.
- Use in activities such as these:

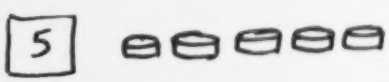
Draw 5 cards and arange in order, smallest to largest

Draw 2 cards. Add or multiply them.

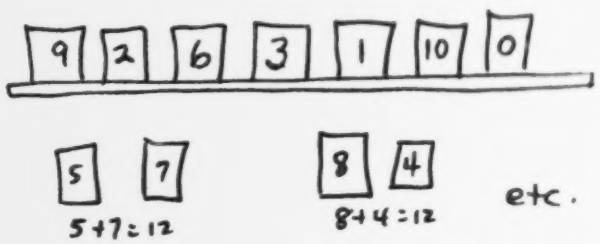
Draw a card and subtract from 10 (or some other number).



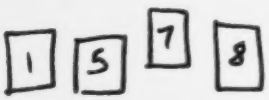
Draw a card and match it with appropriate number of counters



Display cards on the chalkrail or table. Children take turns choosing pairs of cards with a certain sum. (If 2 is the sum, children choose pairs such as 6 and 6, 5 and 7, 3 and 9, etc.)

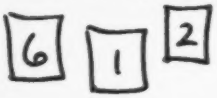


Choose 3 or 4 cards. Form a numeral. Say the numeral. Or choose cards and let one child specify a numeral for a second child to arrange. If a first child draws 8, 2, and 5, a second child might ask for five hundred twenty eight.



"one thousand five hundred seventy eight"

"Make one hundred twenty six from these."



Board Games

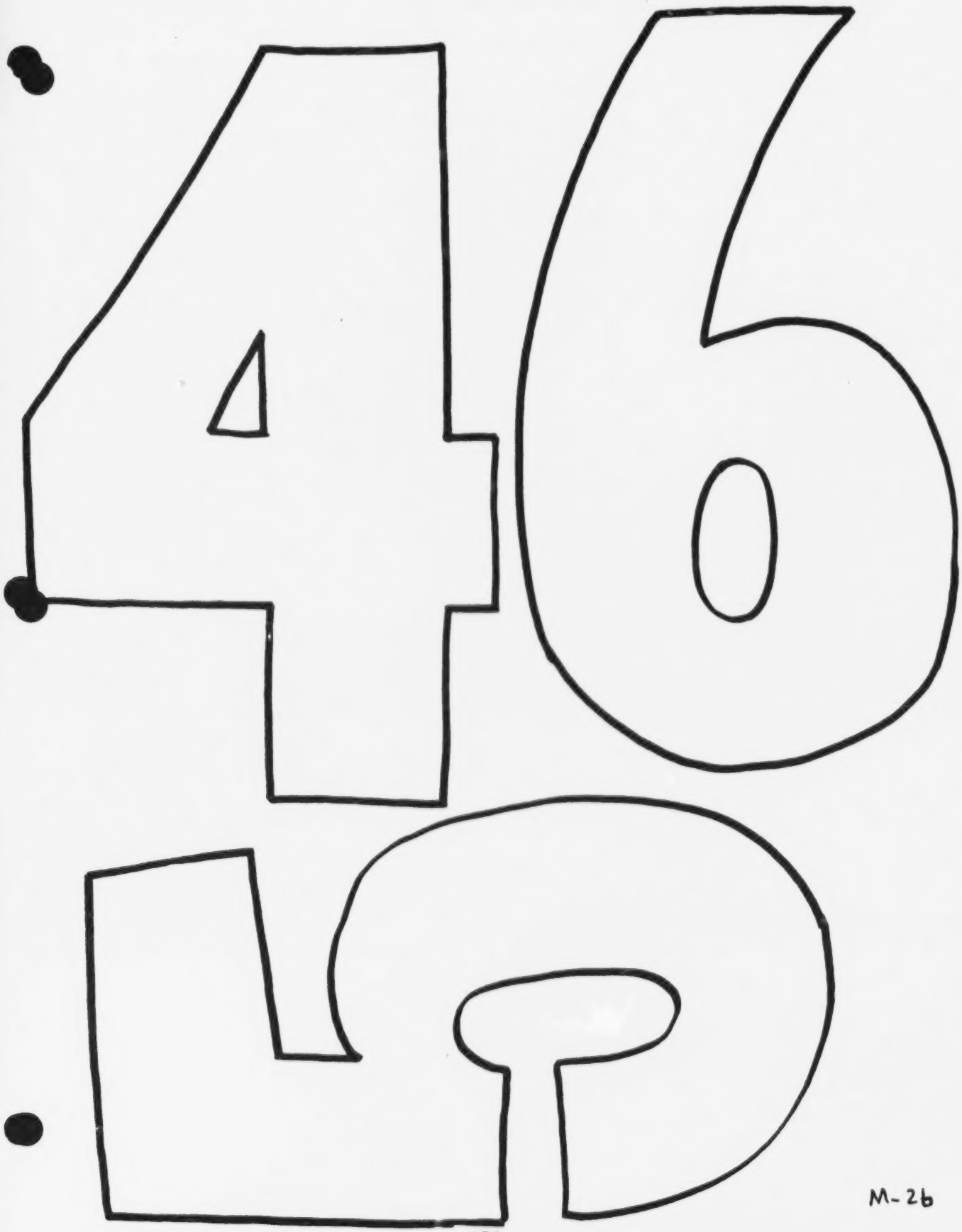
- Use for reinforcement of any skill.
- Materials: Posterboard for game boards (Patterns M-4), notecards or construction paper, markers, golf tees for spinners, large paper clips, brads.
- Create a game board using a suggested design. Make a deck of problem cards appropriate for what children are studying (or children could even make their own using facts they need practice on). You might make some "BONUS" and a few "Miss-a-Turn" cards. Make a spinner for your board game.
- Children work in a small group taking turns trying to answer problems and moving their pieces around the gameboard.

Flashcards

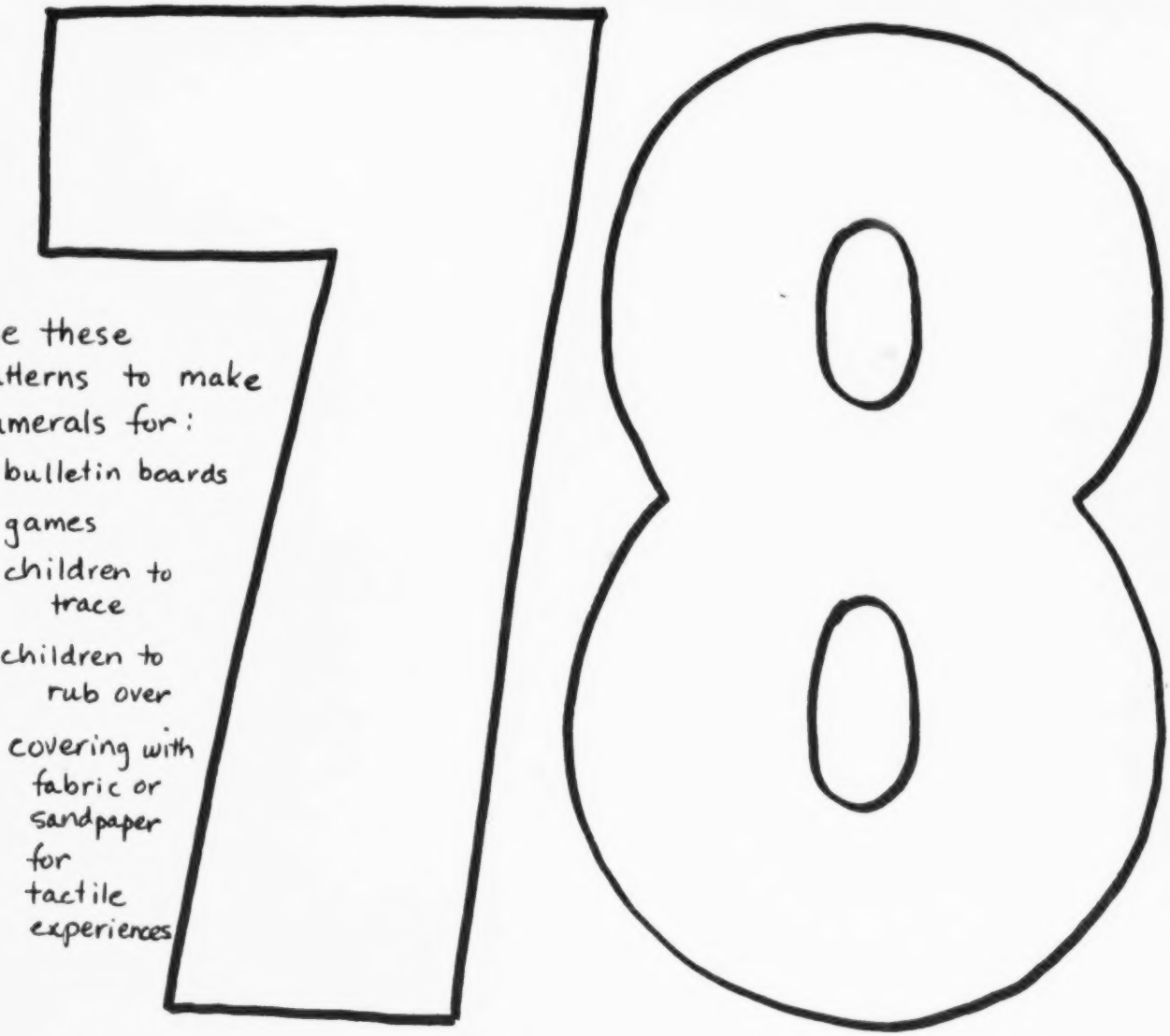
- Use for individual practice on basic facts.
- Materials: Construction paper, old computer cards, or minilla-folder weight paper.
- Make several sets of flashcards. Some teachers will want to make samples, and let the children make their own flashcards for practice on facts each needs to master. (Patterns M-5)
- Children use flashcards individually or with a partner.

NUMERAL PATTERNS

M-2a

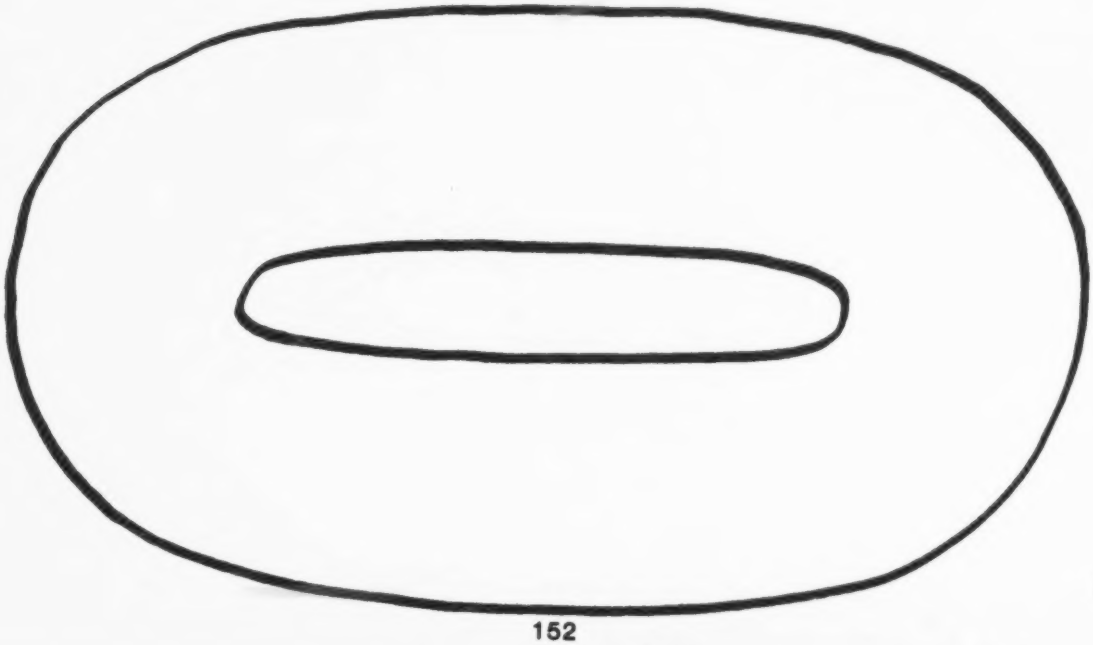


M-26



Use these patterns to make numerals for:

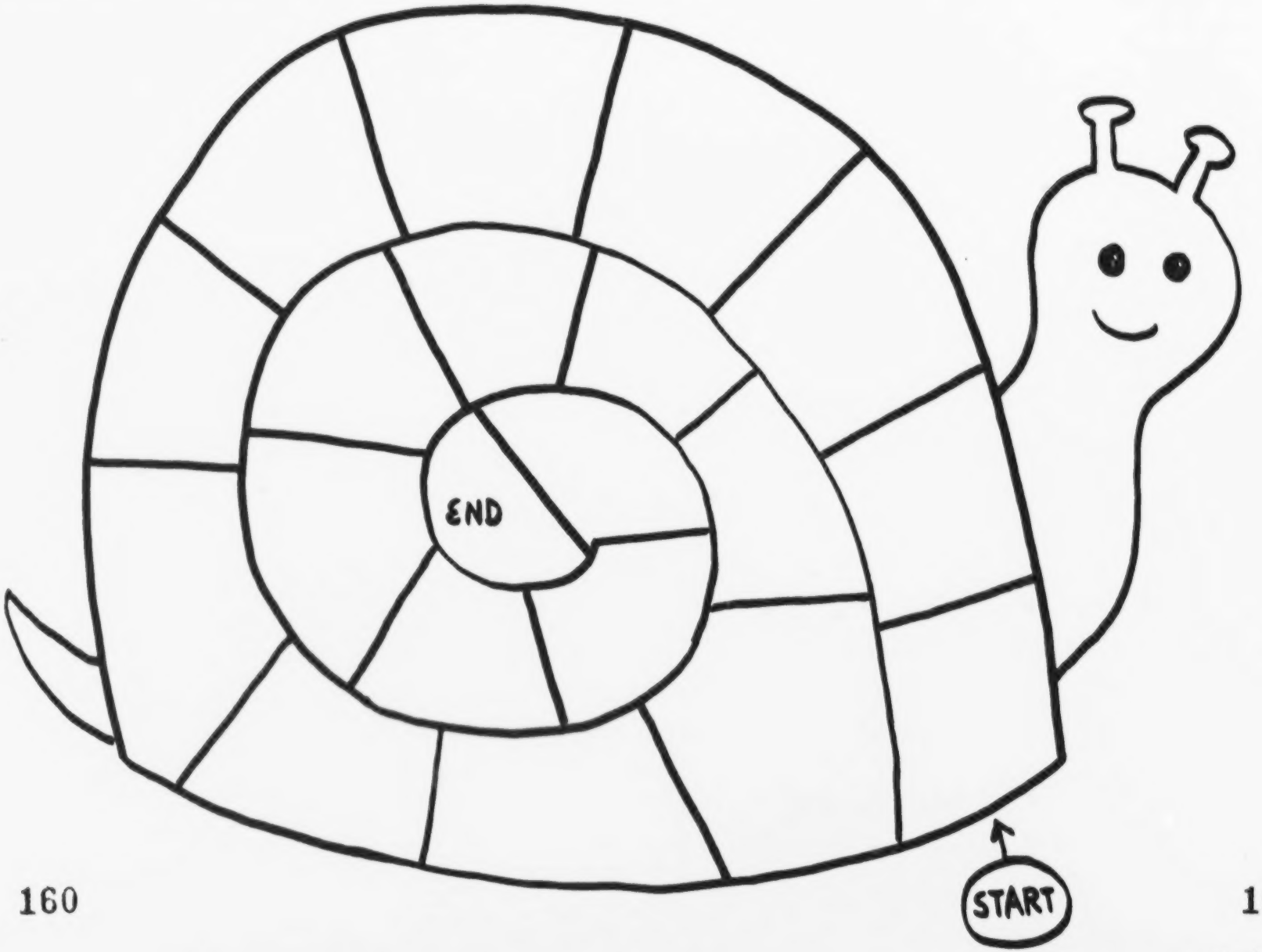
- bulletin boards
- games
- children to trace
- children to rub over
- covering with fabric or sandpaper for tactile experiences



M-2c

Foot print Pattern

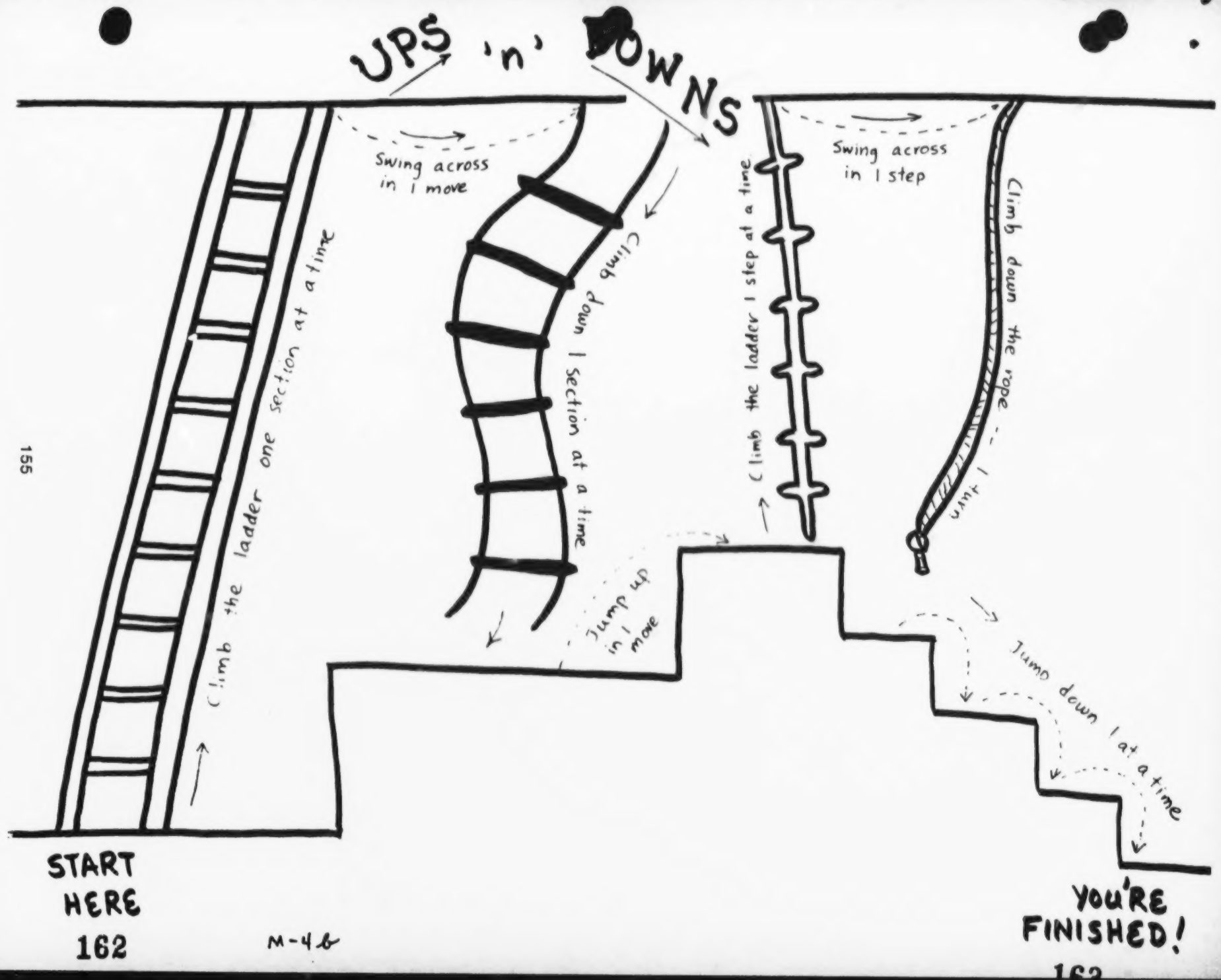
SNAIL'S PACE

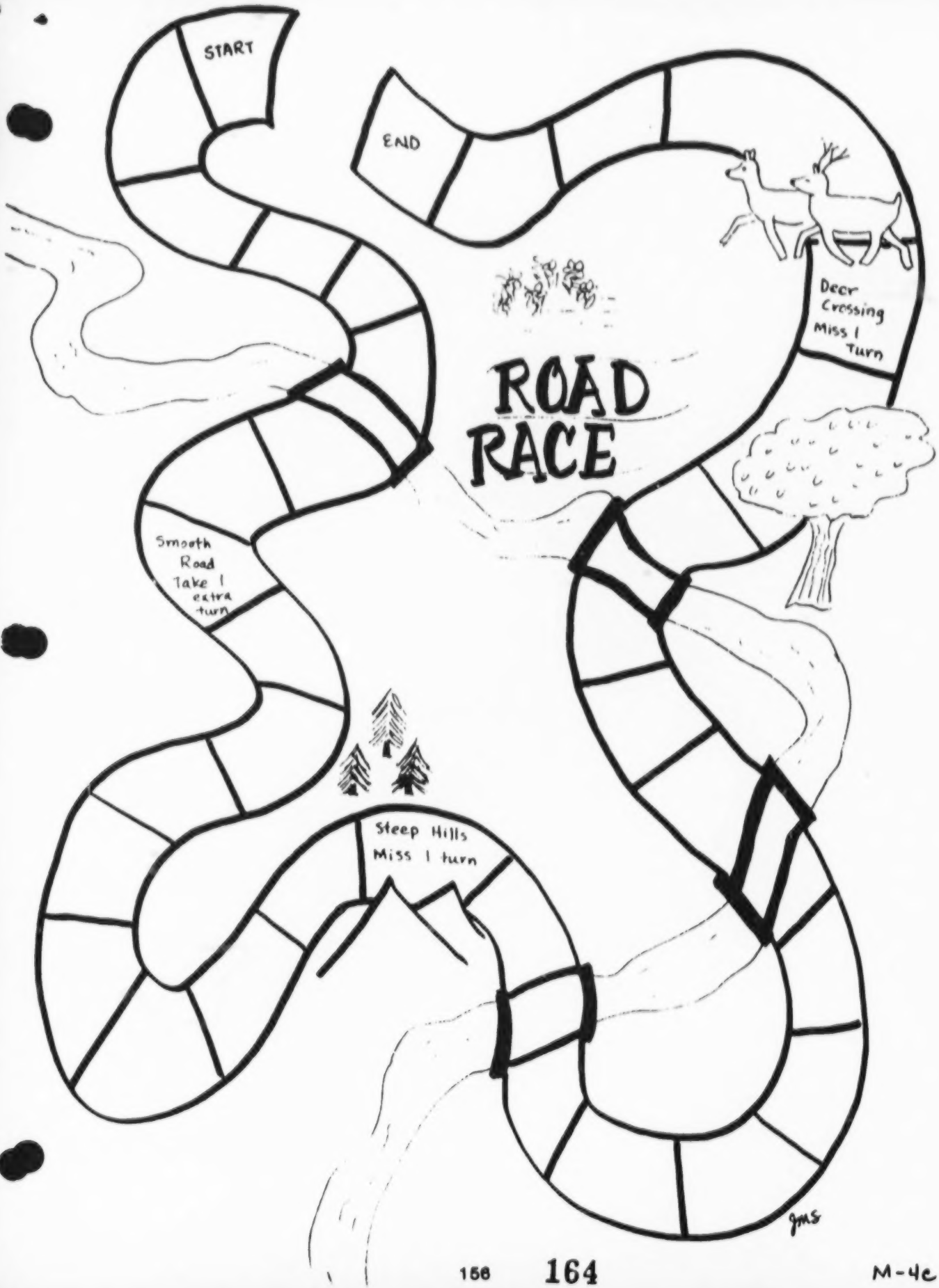


160

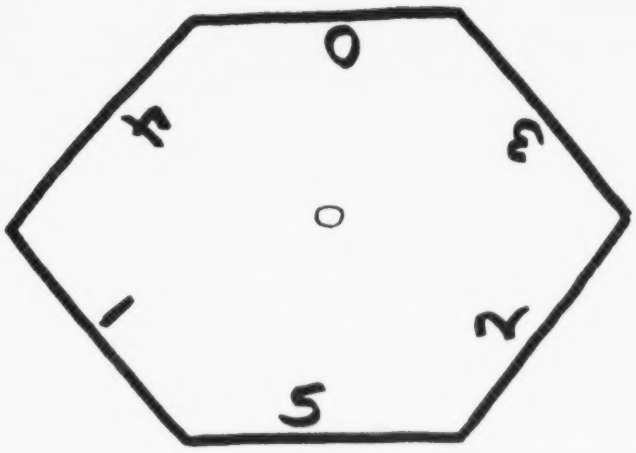
161

M-4a

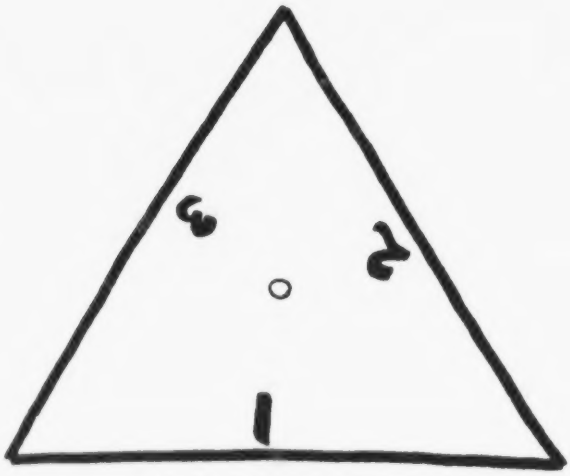

UPS 'n' DOWNS
Swing across in 1 move
Climb down 1 section at a time
Climb the ladder one section at a time
Swing across in 1 step
Climb down the rope -- 1 turn
Climb the ladder 1 step at a time
Jump up in 1 move
Jump down 1 at a time
START HERE 162
YOU'RE FINISHED!
M-4b


START
END
ROAD
RACE
Deer
Crossing
Miss 1
Turn
Smooth
Road
Take 1
extra
turn
Steep Hills
Miss 1 turn
gms

BOARD GAME patterns



Golf Tee Spinner



Golf Tee Spinner

Golf Tee Spinner

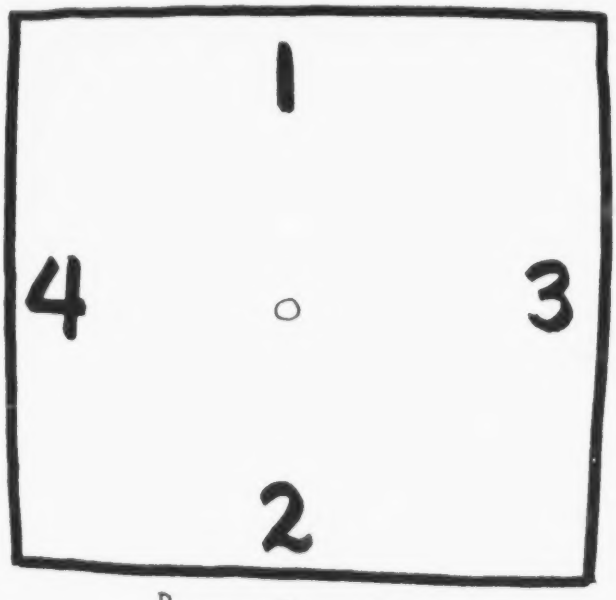
Duplicate and cut out a spinner shape from above. Insert a golf tee through the hole in the middle. The children take the number of moves shown on the side that hits the floor or table top

"2 moves"

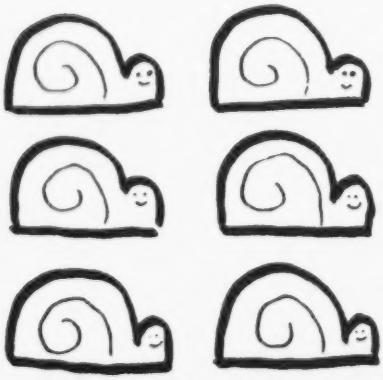
"1 move"

Paper Clip Spinner

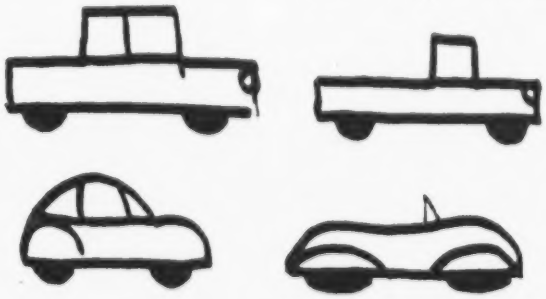
Attach a brad through the hole in the center. Put a paper clip on the brad. Leave it loose enough to spin freely.



Paper Clip Spinner



Markers for "Snail's Pace"

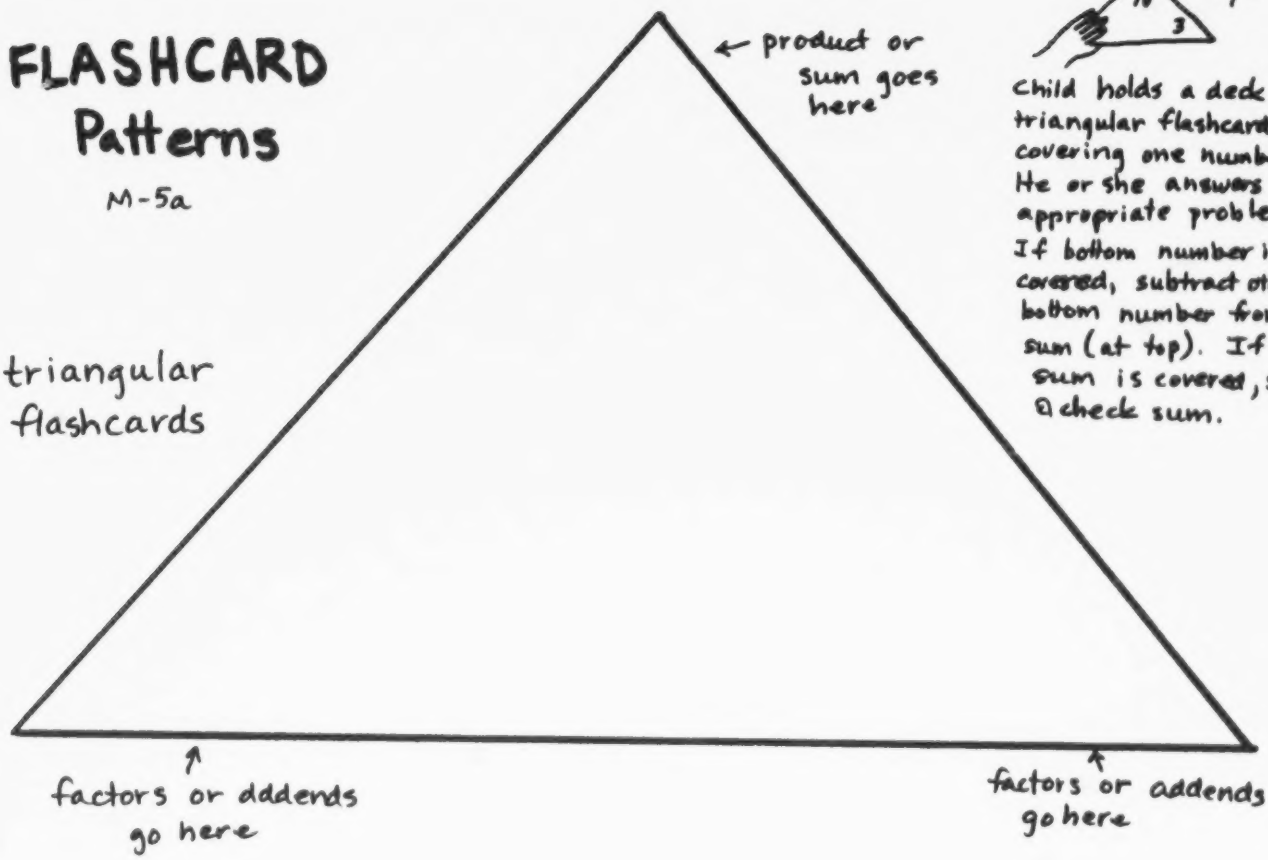


Markers for "Road Race"

Use pennies or tiny plastic animals for markers for "Ups 'n' Downs"

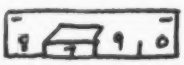
FLASHCARD Patterns

M-5a



Child holds a deck of triangular flashcards covering one number. He or she answers appropriate problems. If bottom number is covered, subtract other bottom number from sum (at top). If sum is covered, say & check sum.

flip-up flashcards



Child says answer, then flips up flap to see answer. Staple top to bottom.

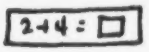
Add 3 to each number.

8 4 9 0

8+3 = 11 4+3 = 7 9+3 = 12 0+3 = 3

more FLASHCARD patterns M5b

Make both horizontal and vertical formats. Put entire problem on the back.



You can make missing addend cards too --

REFERENCES

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Riedesel, C. Alan. Teaching Elementary School Mathematics. Englewood Cliffs, NJ: Prentice-Hall, Inc., 1980. (This book features use of simple materials for concept development as well as ideas for reteaching topics and presenting topics in unique ways.)

Troutman, Andria P., and Litchtenberg, Betty K. Mathematics: A Good Beginning. (Organized for concept and skill development, the book describes hundreds of learning activities and valuable ideas for diagnosis and remediation of common problems.)